

## Lecture 5

# Housing Markets and Kidney Exchange

Reminder about starting recording

# Housing Market

[Shapley and Scarf, *JME* 1974]

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[Shapley and Scarf, *JME* 1974]





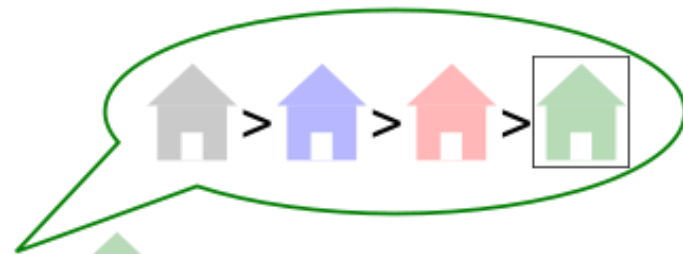
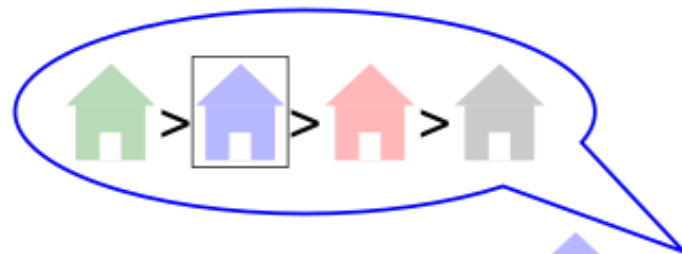
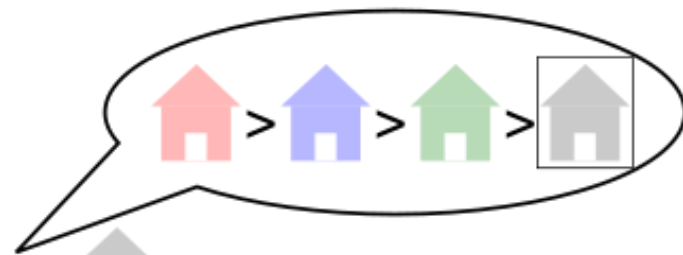
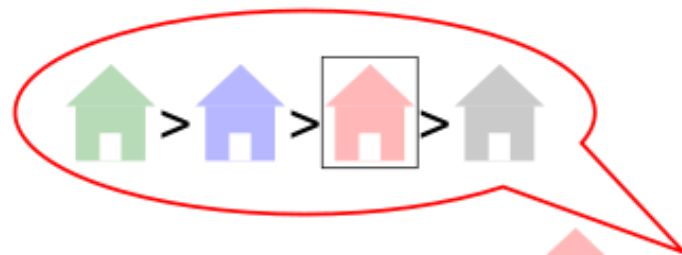
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[Shapley and Scarf, *JME* 1974]



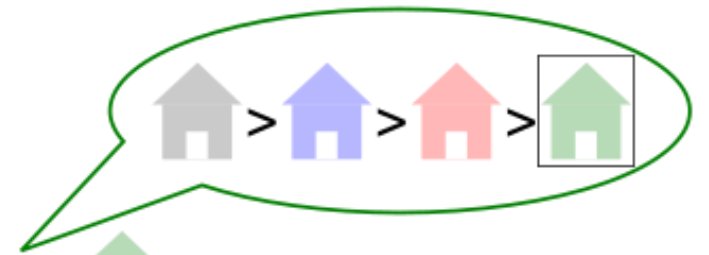
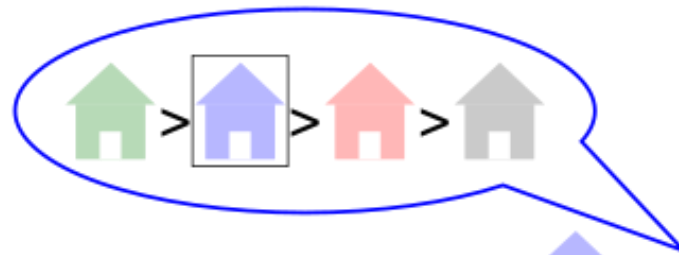
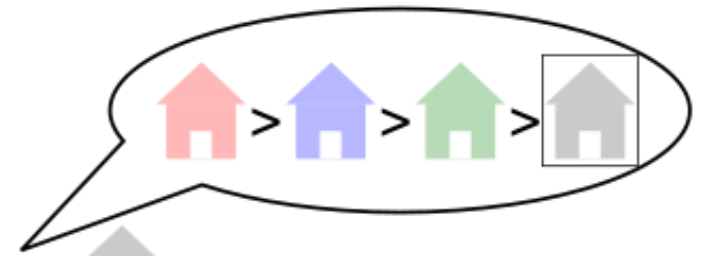
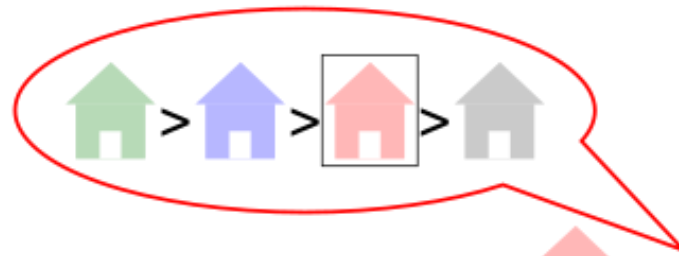
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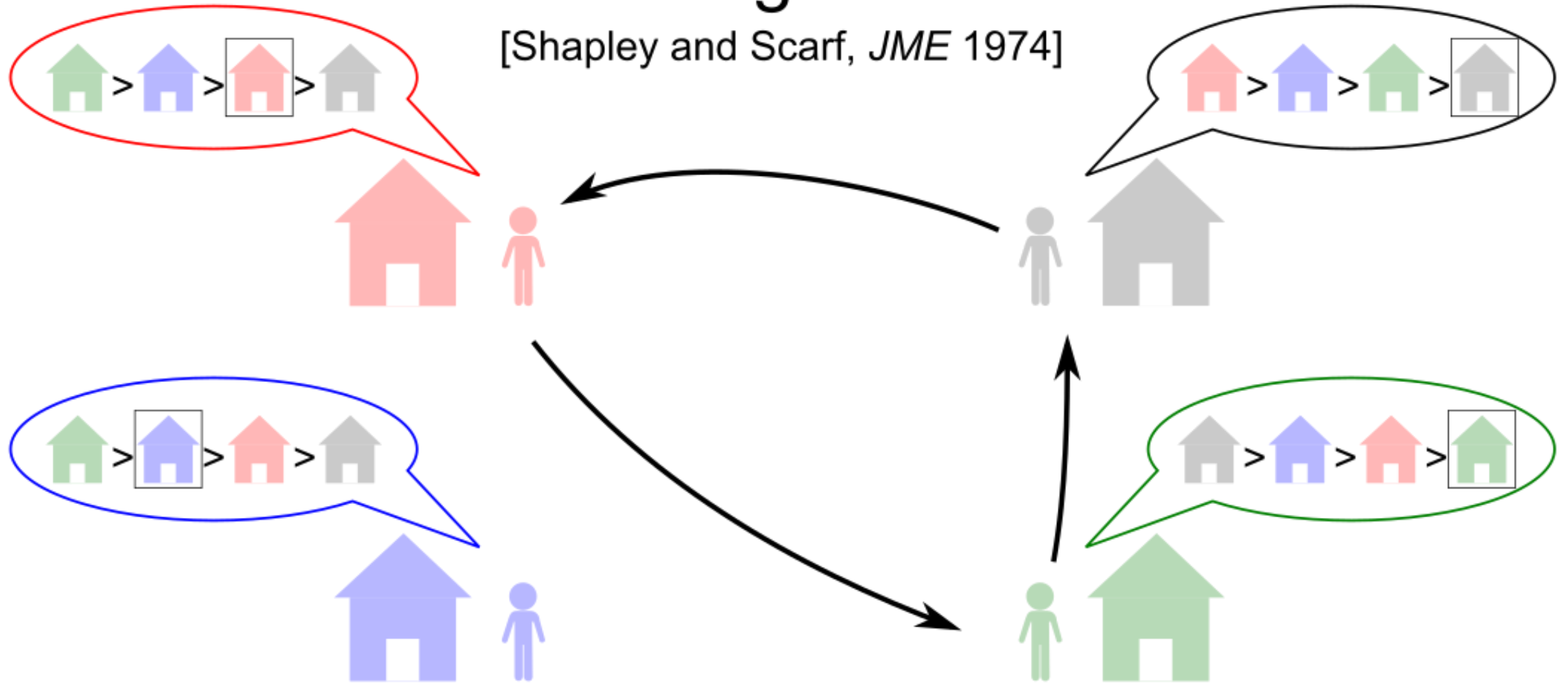
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Can't use money. Only way to make agents happier is via *exchanges*.

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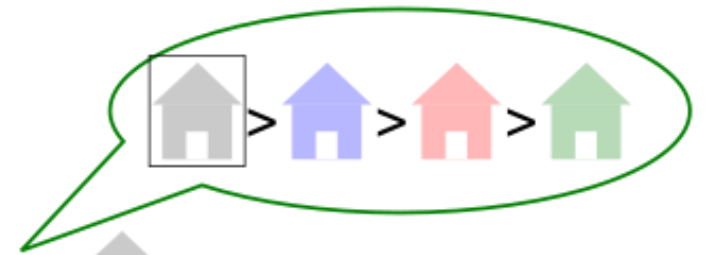
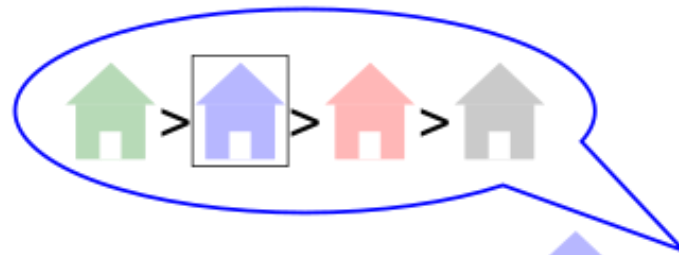
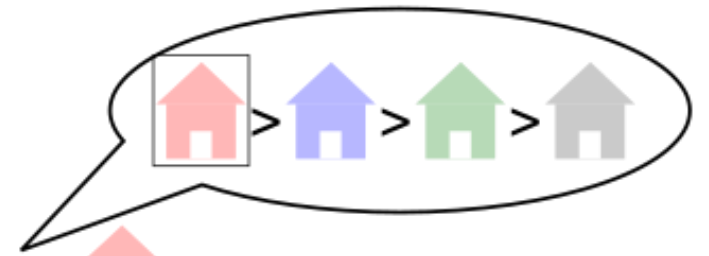
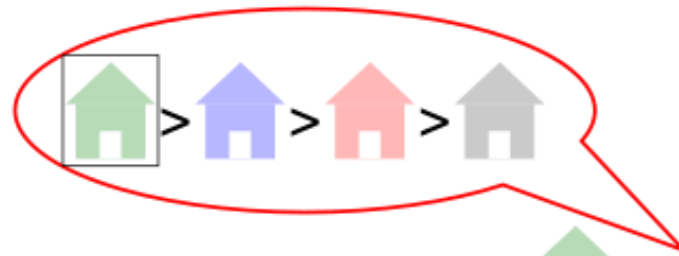
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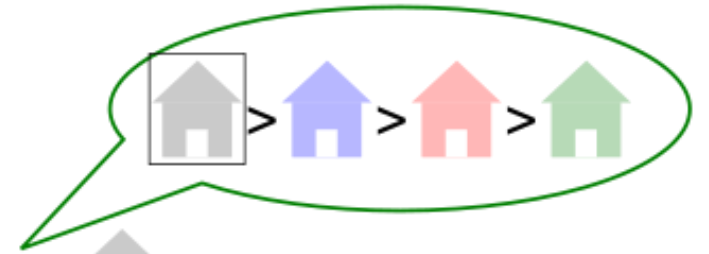
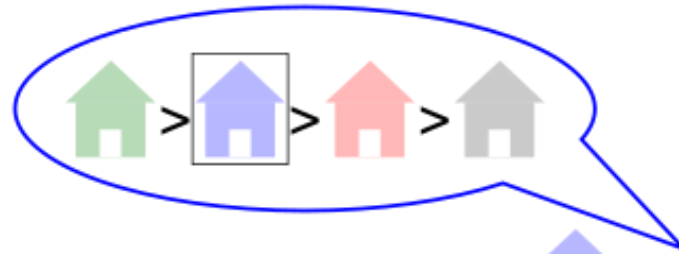
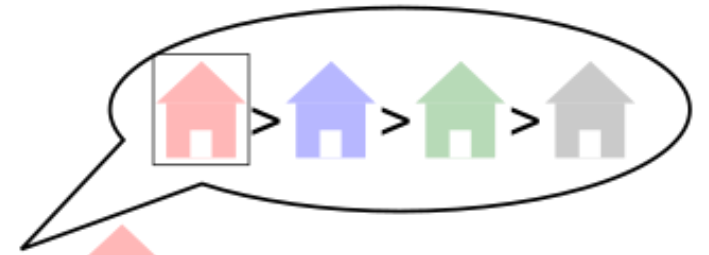
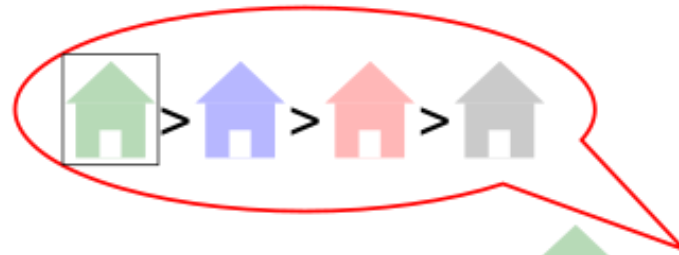
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# Housing Market

[Shapley and Scarf, *JME* 1974]



Is there a way to exchange houses to make the agents "maximally happy"?

# Top-Trading Cycle Algorithm (TTCA)

[Shapley and Scarf, *JME* 1974 (attributed to David Gale)]

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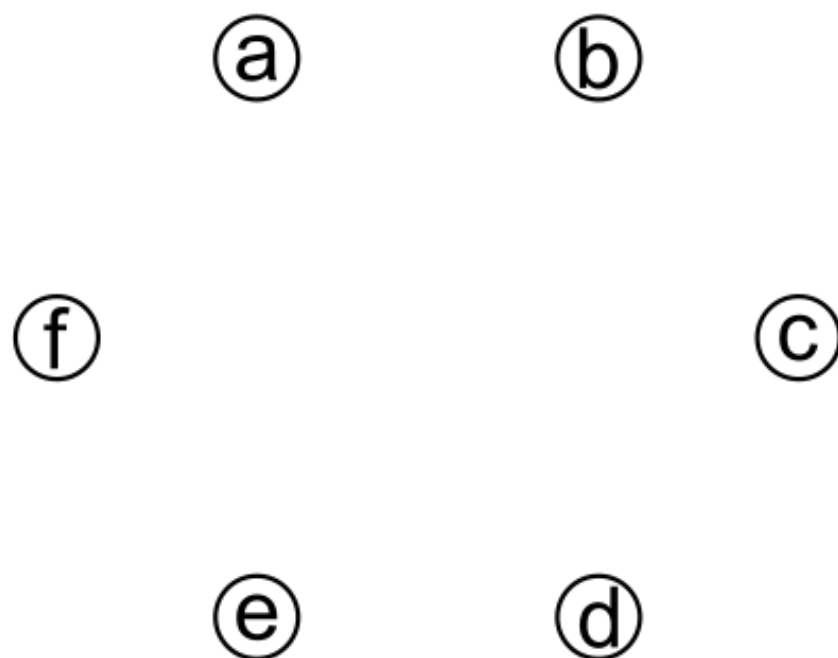


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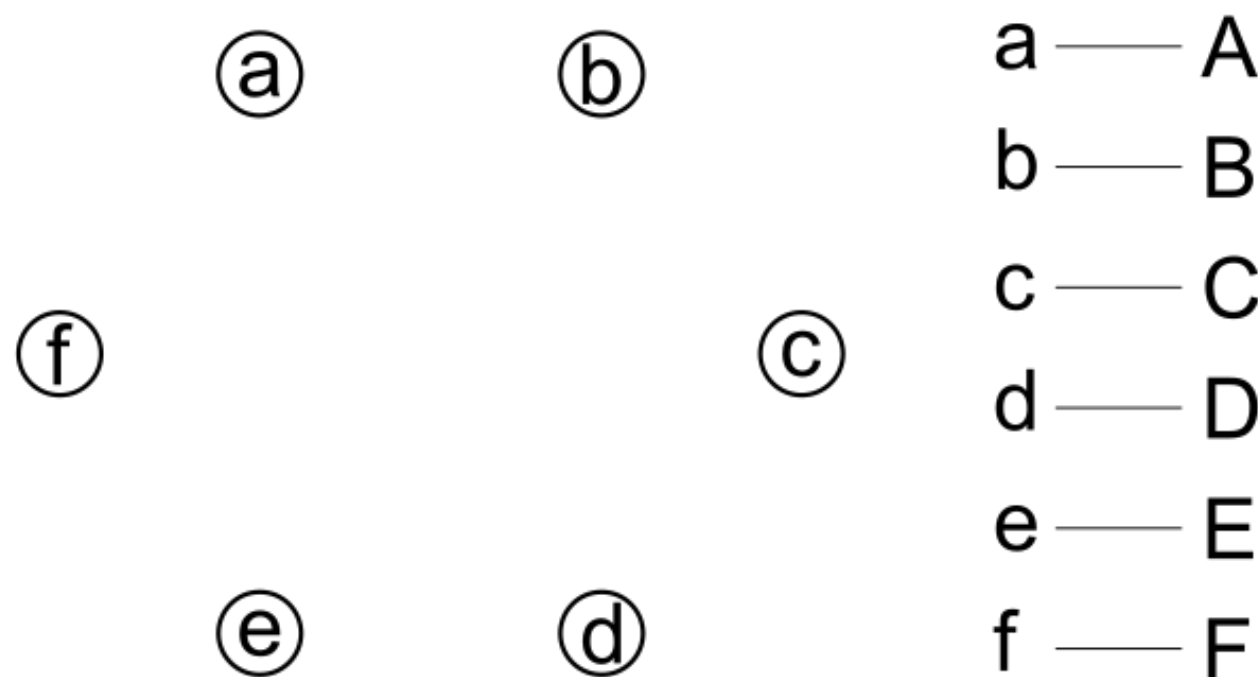


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	a	b	c	d	e	f				
1.	B	F	A	E	C	F	(a)	(b)	a — A	
2.									b — B	
3.				(f)				(c)	c — C	
4.									d — D	
5.									e — E	
6.					(e)			(d)	f — F	

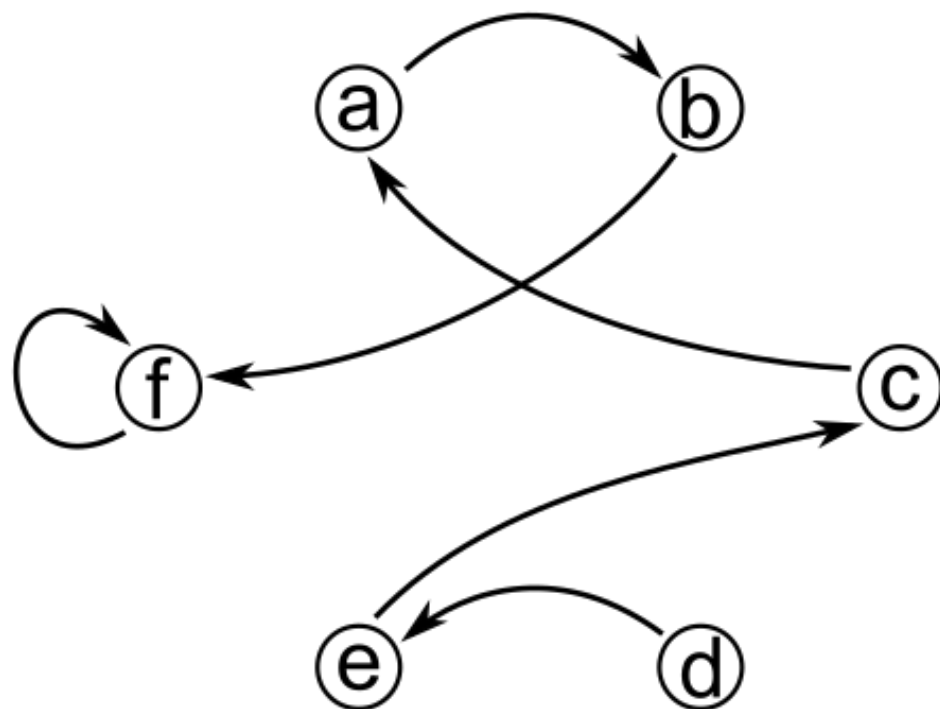
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a	—	A
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c	—	C
d	—	D
e	—	E
f	—	F

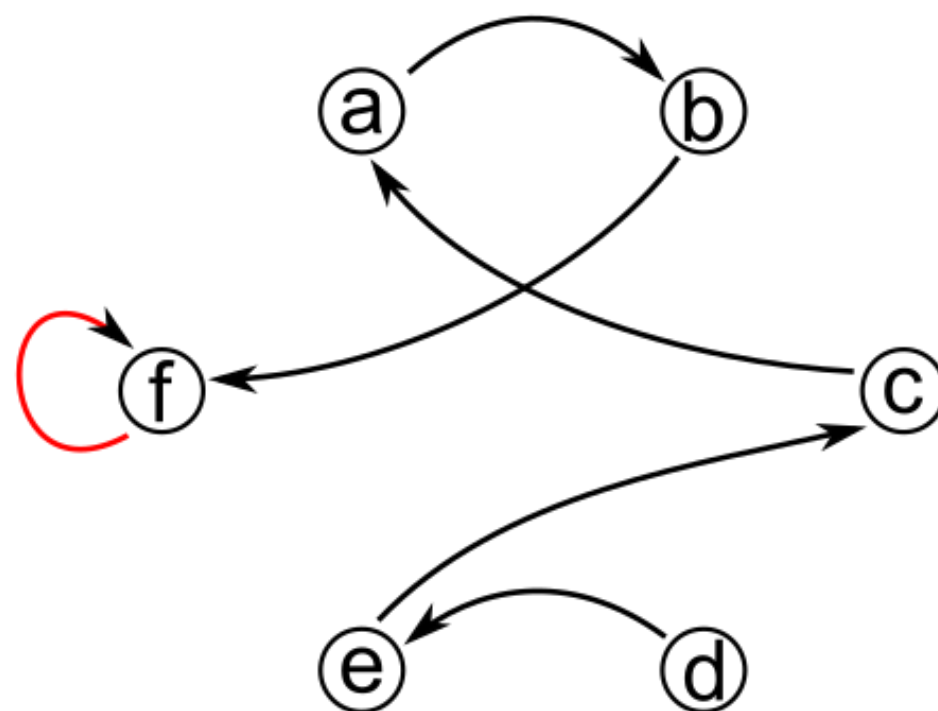
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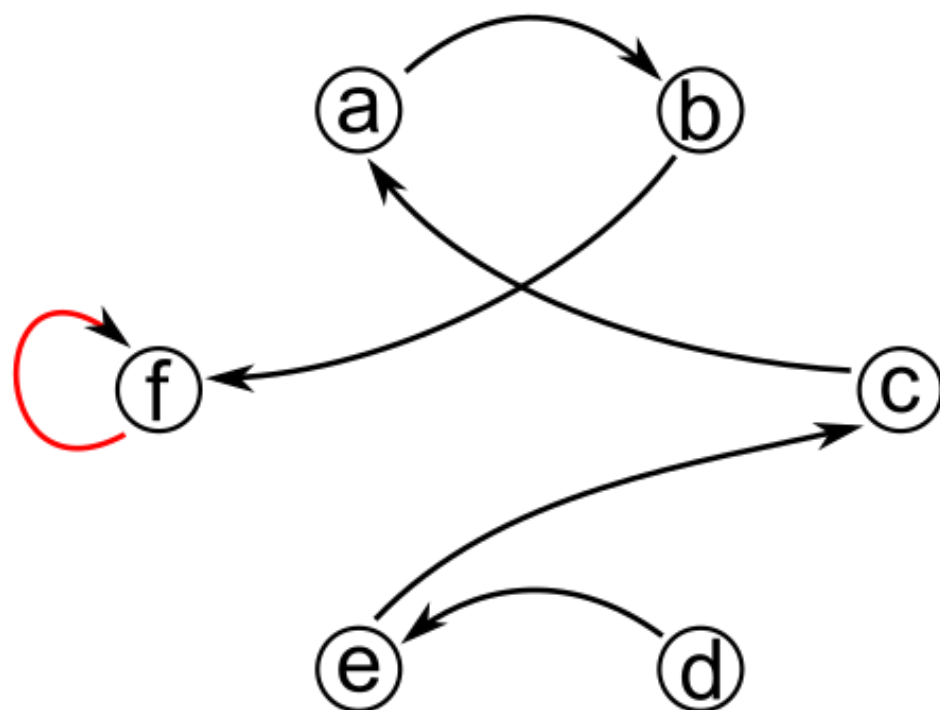
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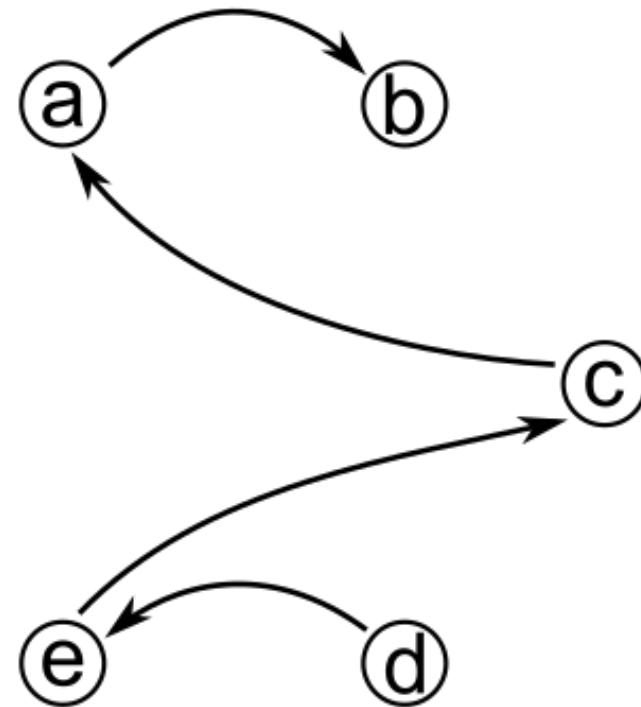
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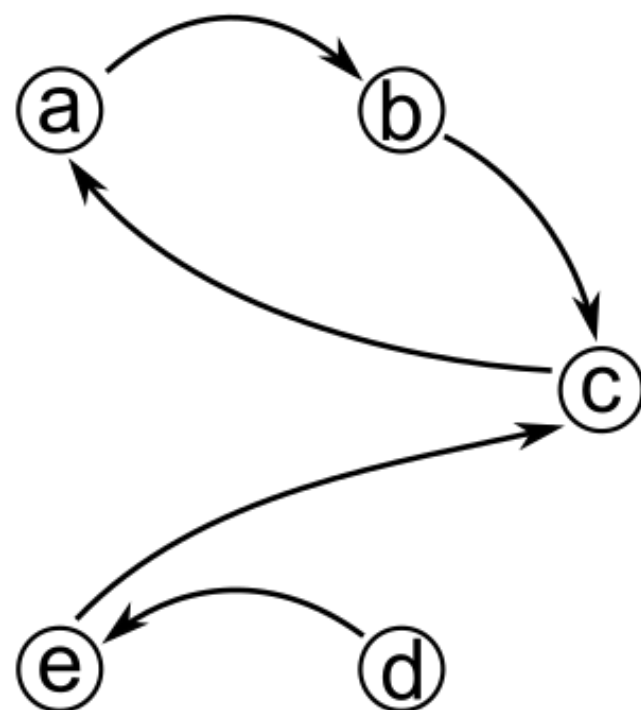
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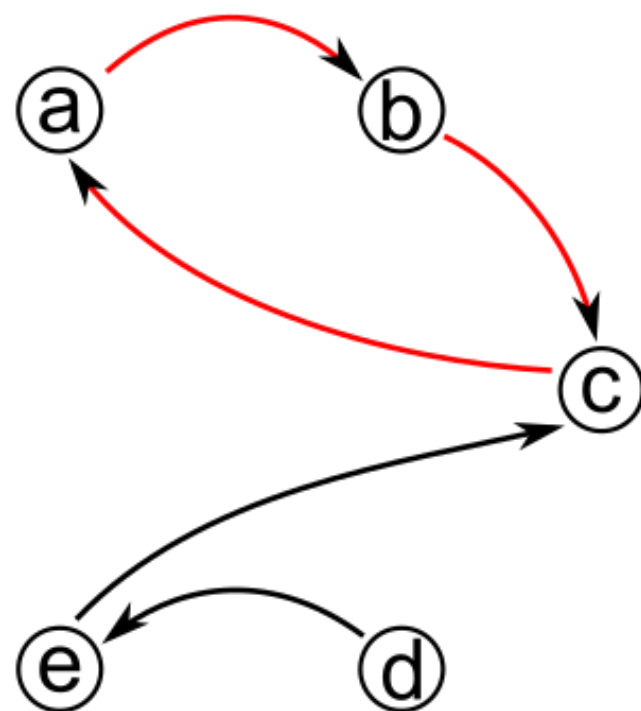
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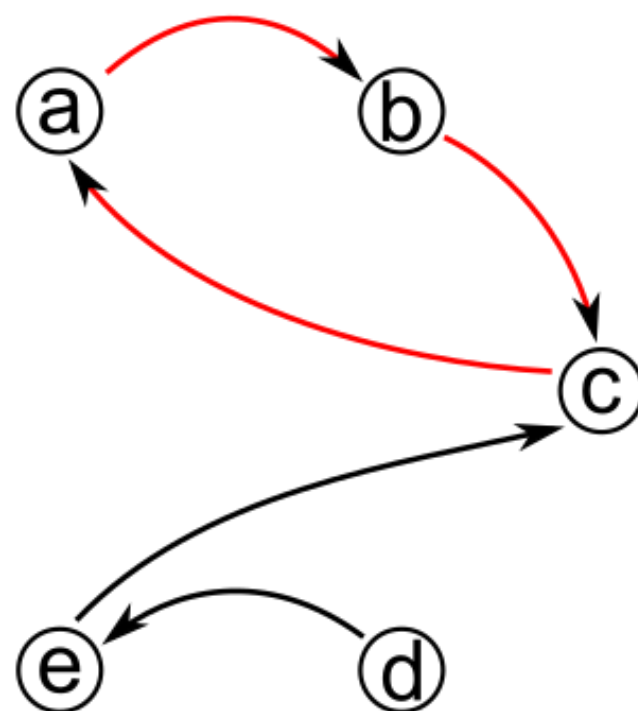
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		d	e
1.	_____	E	
2.			
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5.			
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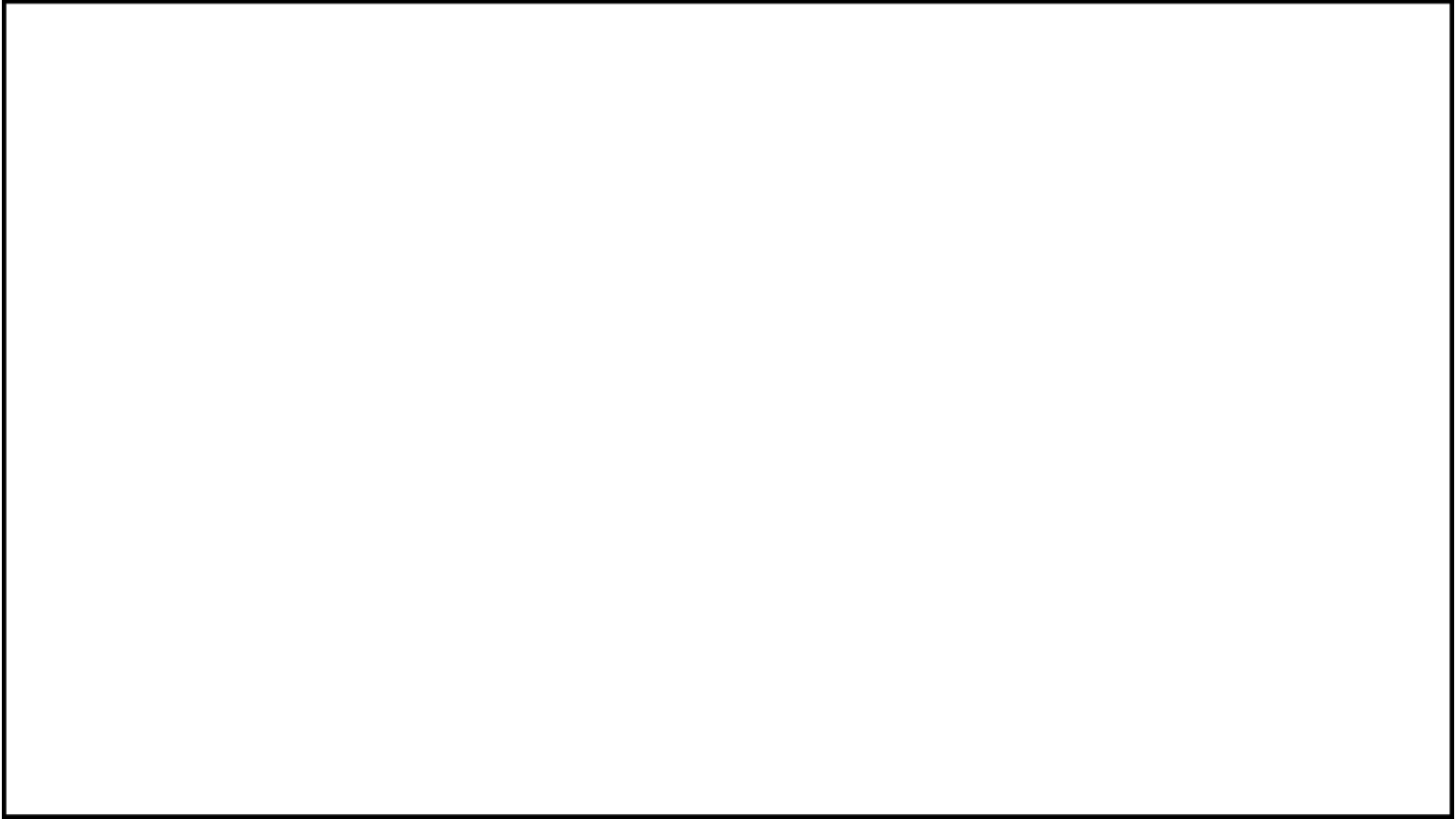
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1.	_____	E	
2.			
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Imagine a graph where the *vertices* are the (remaining) agents and the *edge*  $i \rightarrow j$  means that agent  $i$ 's favorite house is owned by agent  $j$ .

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[Roth, *Econ. Letters* 1982]

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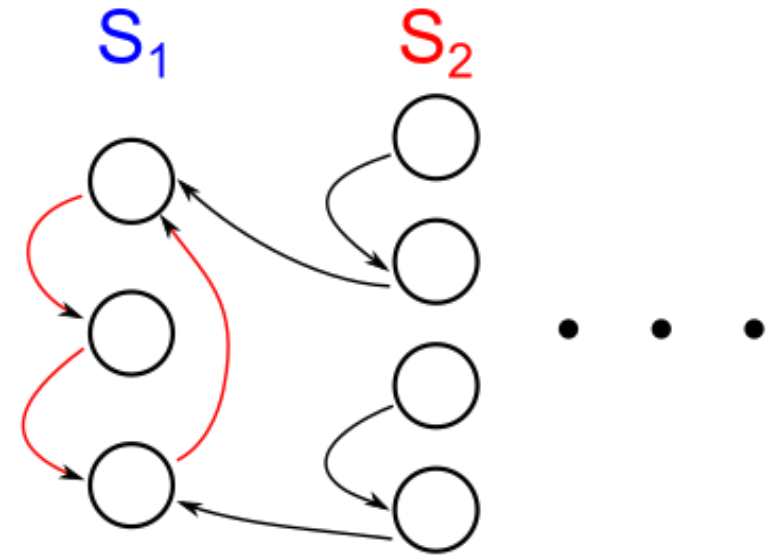


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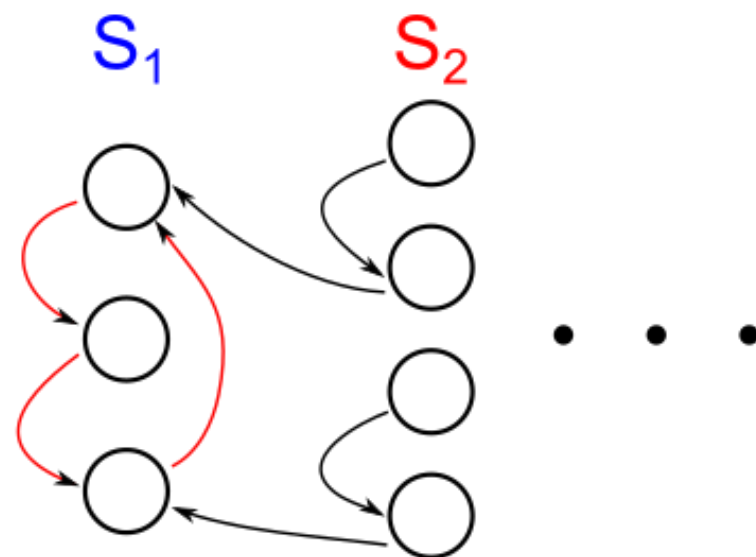
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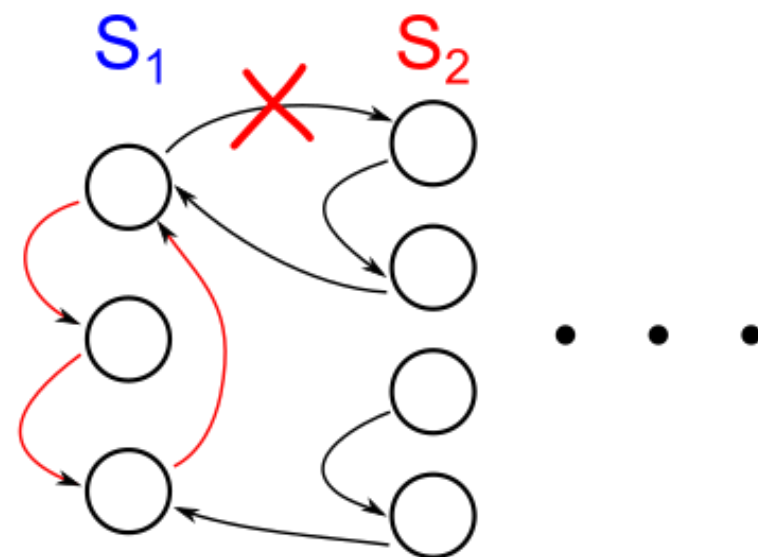
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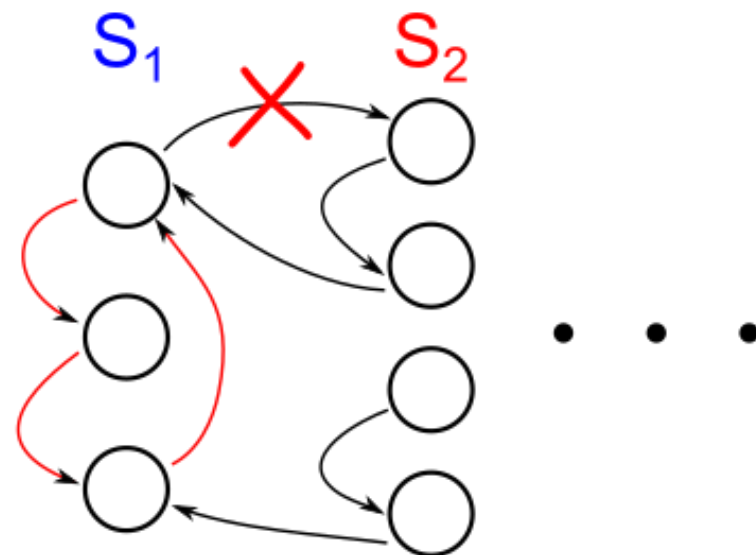


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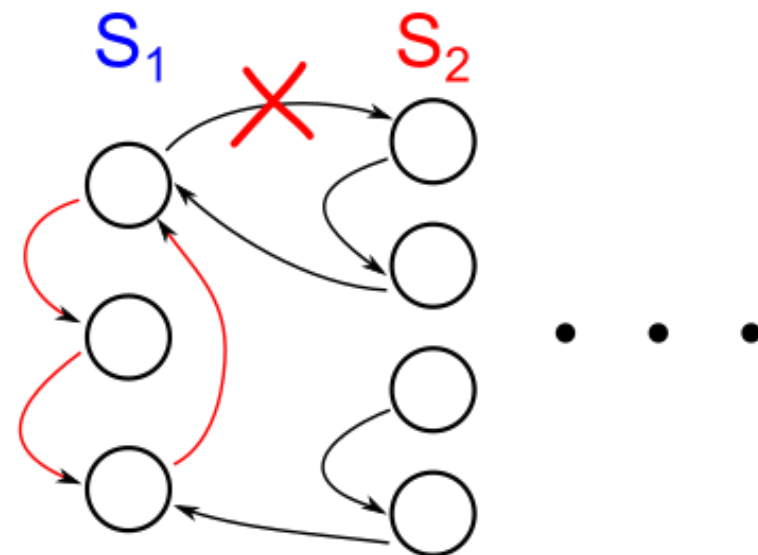
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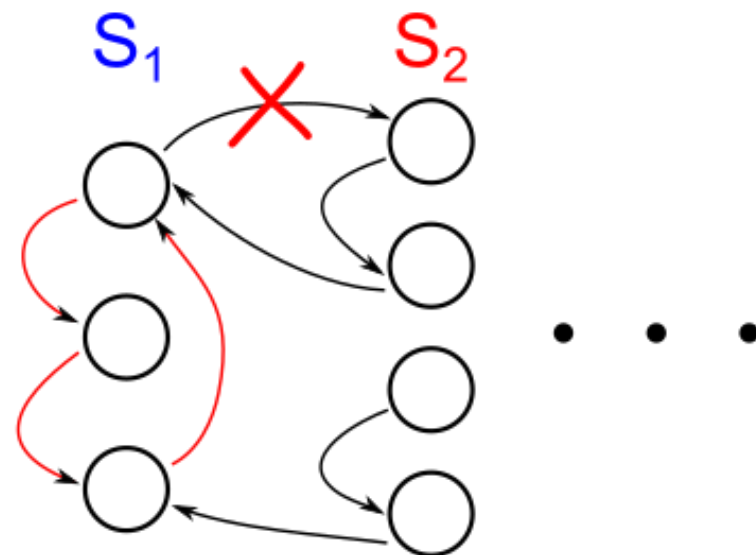
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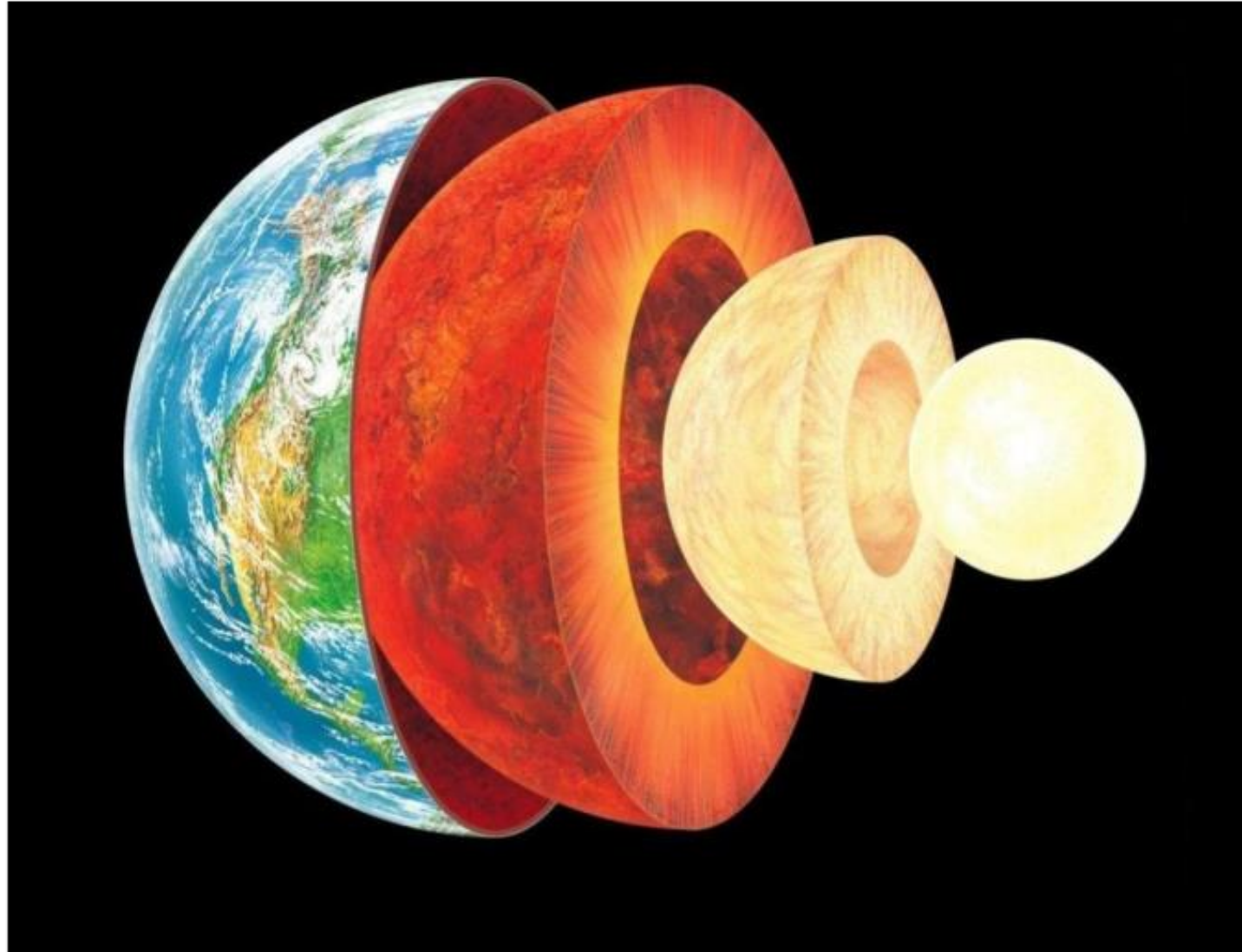
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# The Core



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An allocation is in the *core* if no subset of agents can break away from it, reallocate houses among themselves, and all be strictly better off.

That is, there is no "blocking coalition".



**TTCA outcome is in the core.**

[Shapley and Scarf, *JME* 1974 (attributed to David Gale)]

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Suppose, for contradiction, that a set **B** of agents can break away from the TTCA outcome and improve by internal reallocation.

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Among the agents in **B**, suppose the first one to be removed during TTCA is agent **b** (and suppose **b** is removed in round  $k$ ).

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In fact, TTCA recovers the **unique** core outcome.

[Discuss on Teams channel or on Piazza]

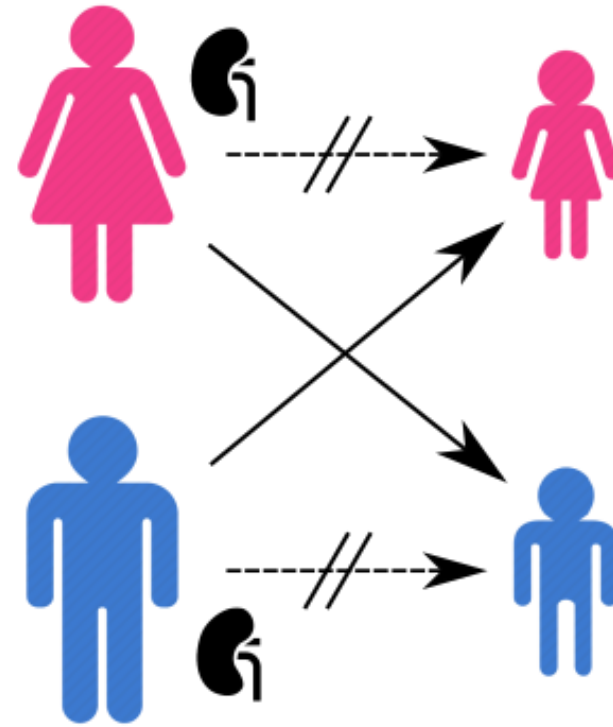








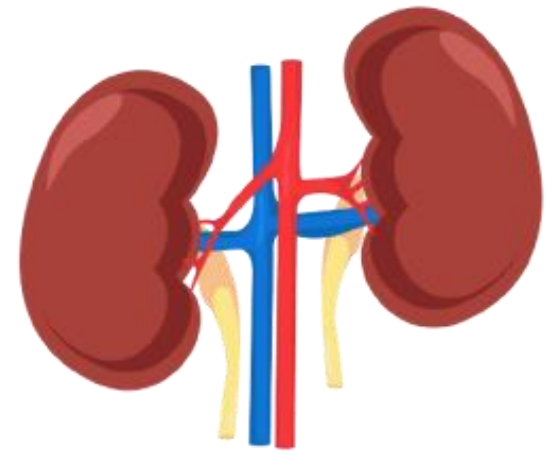
# Application of TTCA



# Kidney Exchange

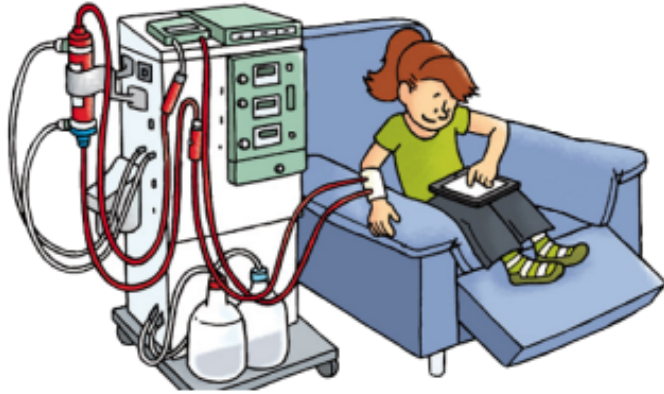
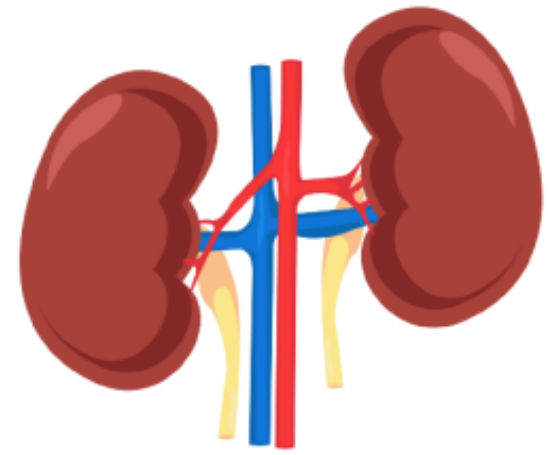
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Many people suffer from kidney failure  
(waiting list in US: ~90,000; in India: ~200,000).



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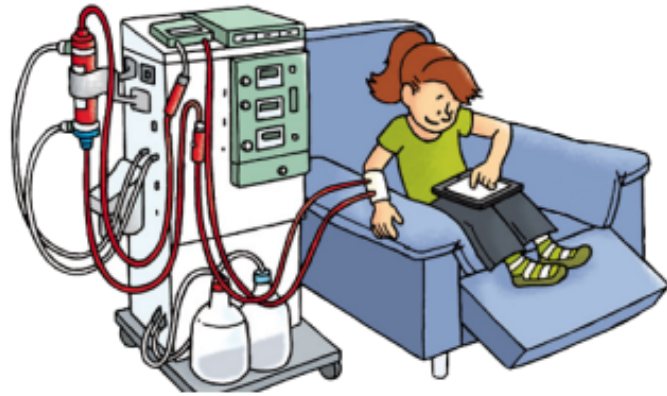
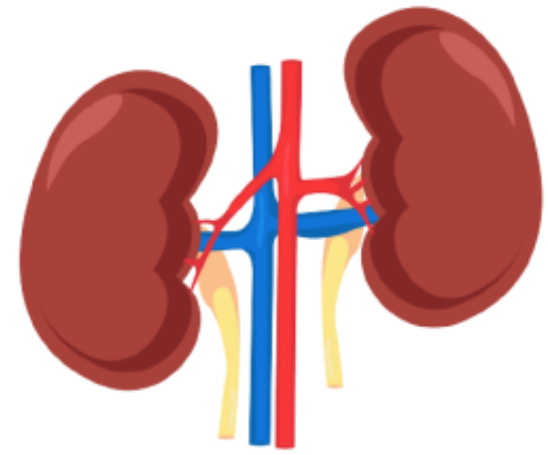
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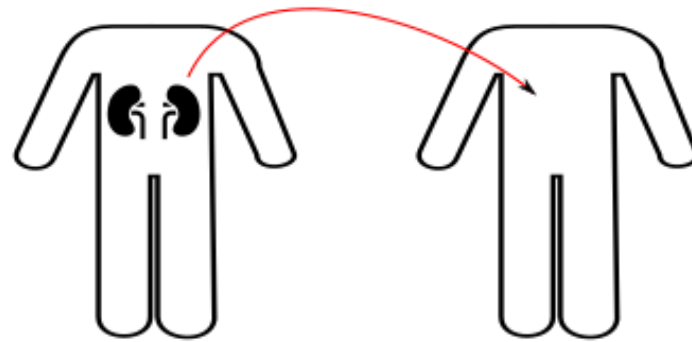
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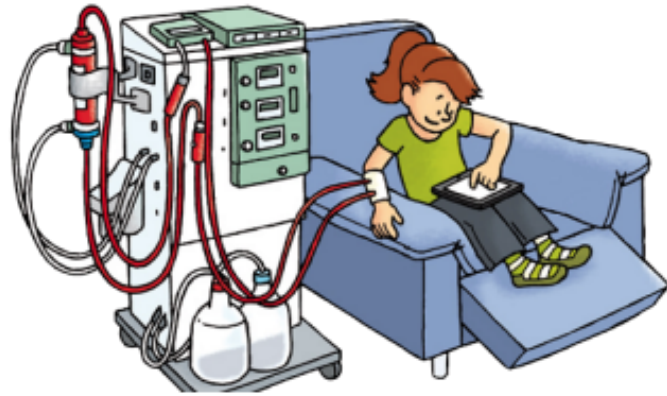
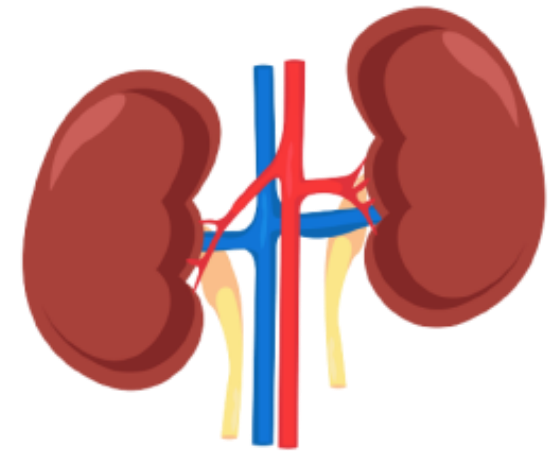


Transplant



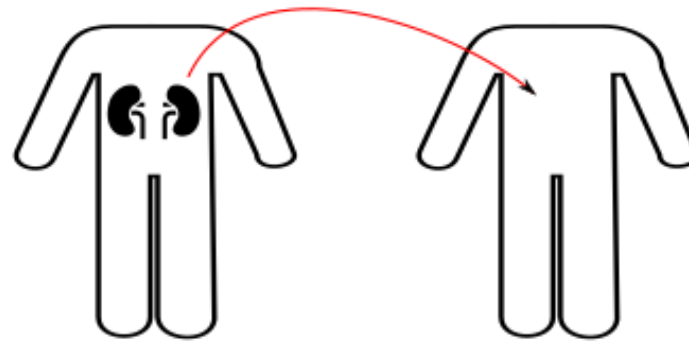
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Poor quality of life  
High cost

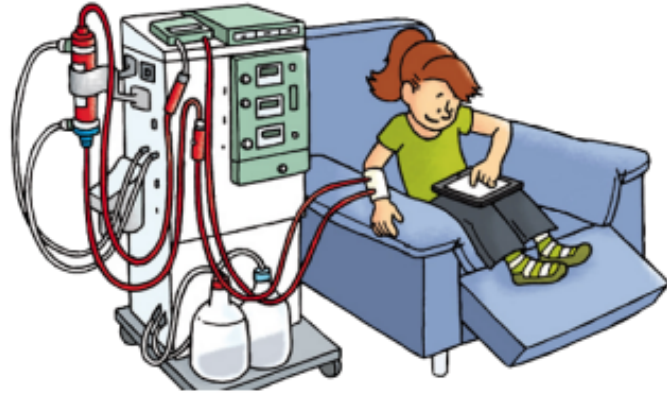
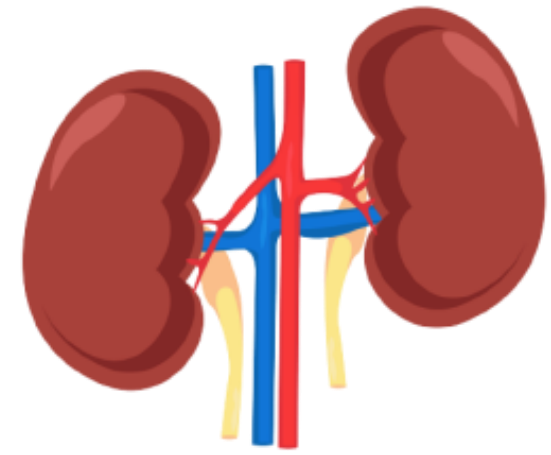


## Transplant

Better life quality  
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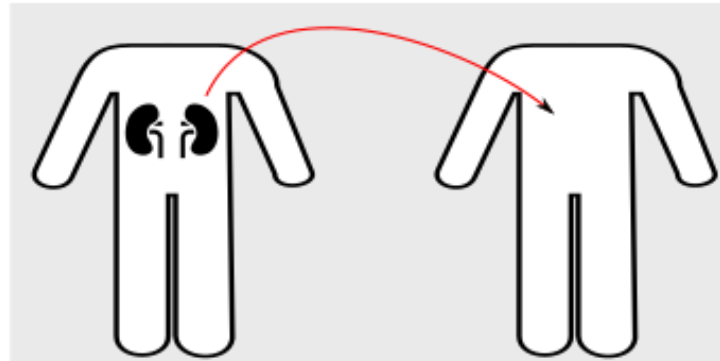
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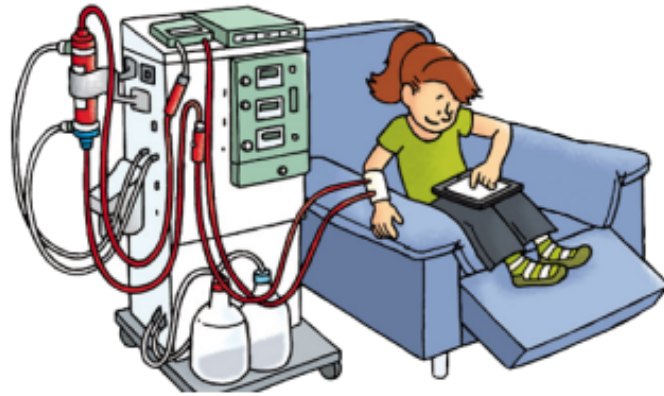
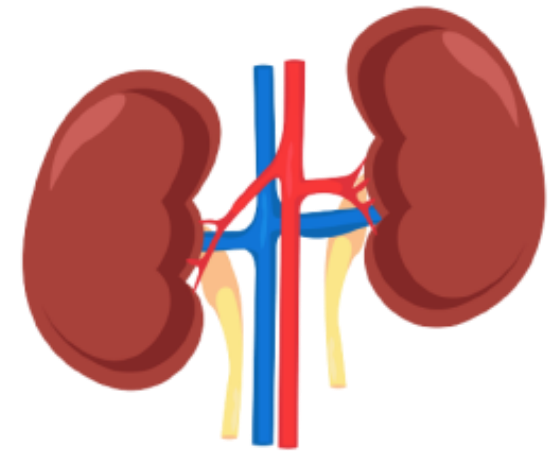
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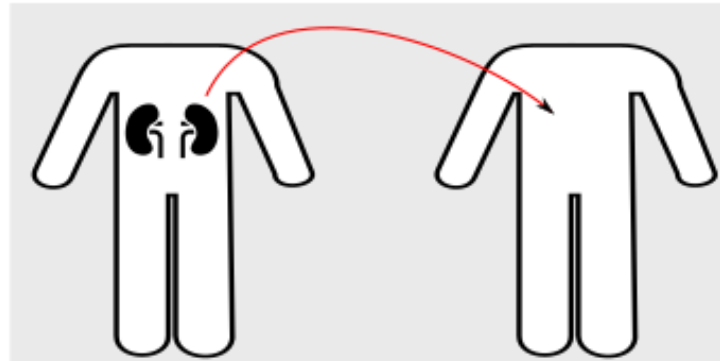
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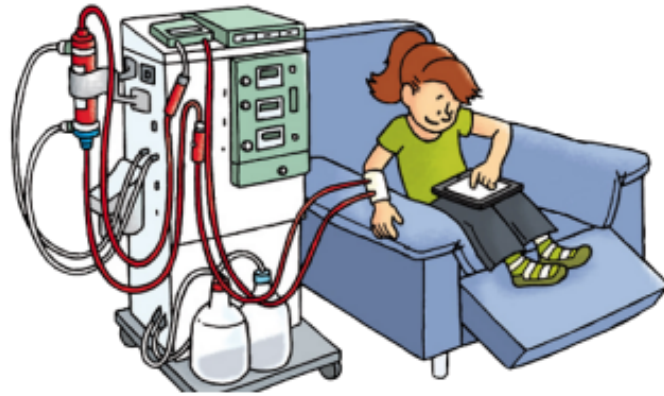
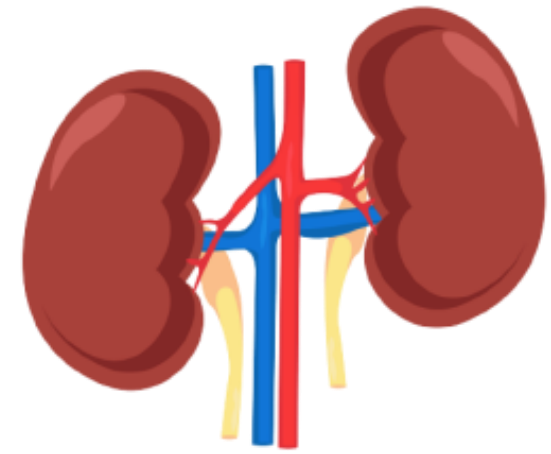
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- From a **deceased donor**

- From a **living donor**

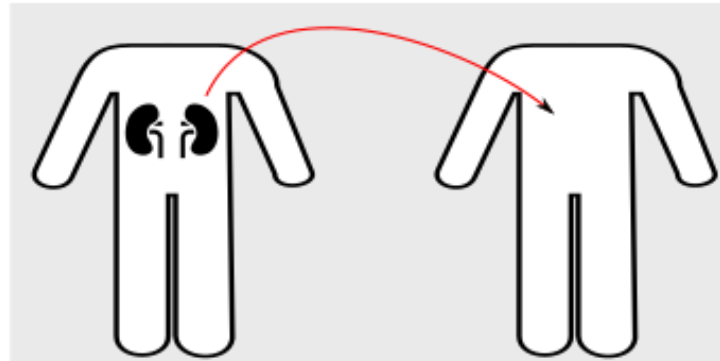
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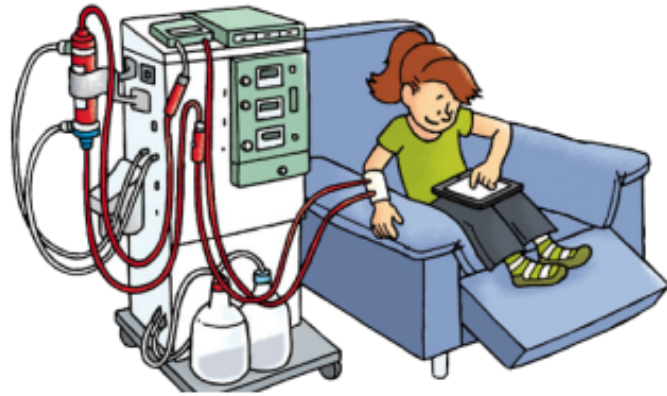
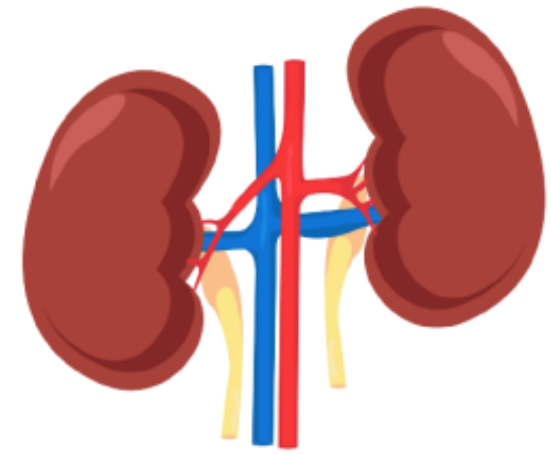
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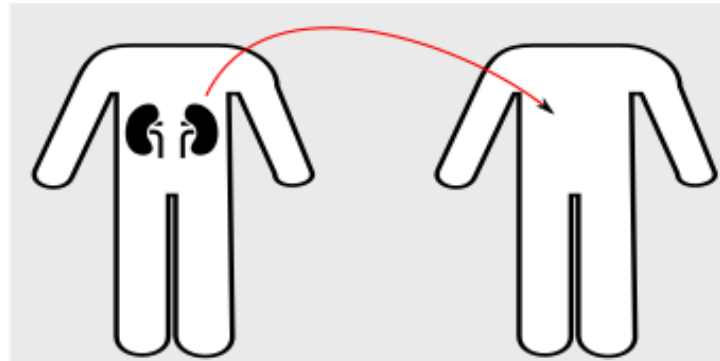
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E.g., first-degree relatives,  
spouse etc.

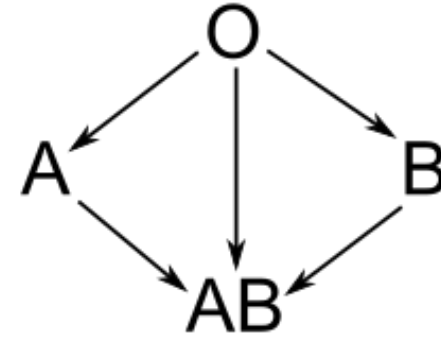
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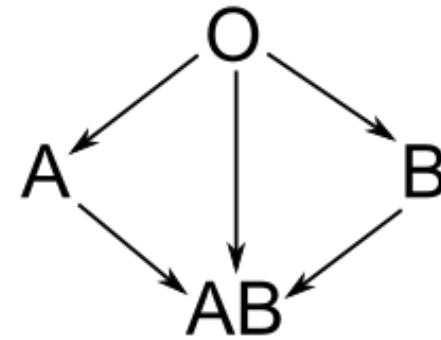
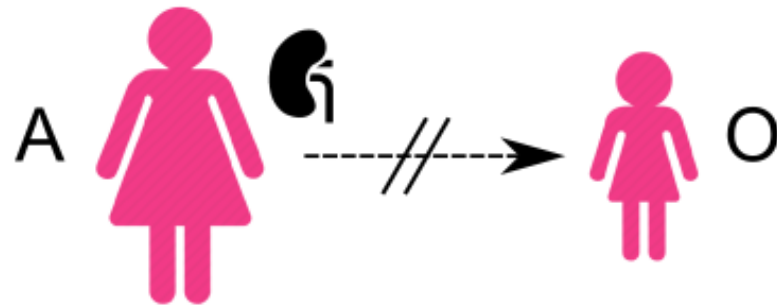
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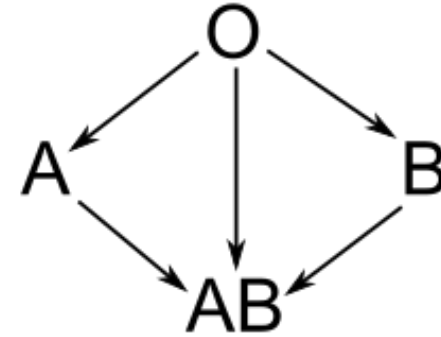
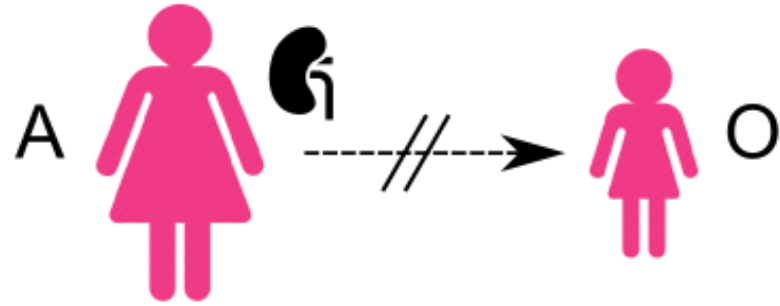
**Blood-type** incompatibility



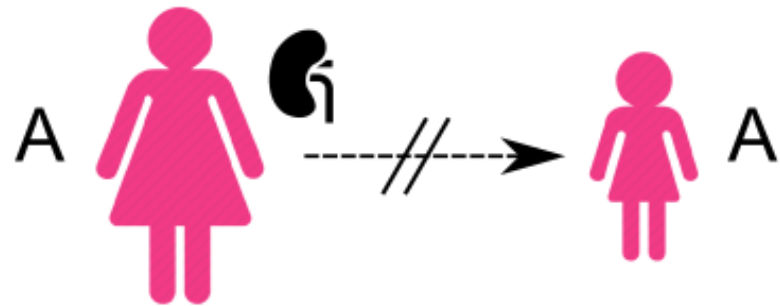
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**Blood-type** incompatibility



**Tissue-type** incompatibility



Sensitized patients with antibodies that will attack a foreign tissue (often due to prior exposure during blood transfusion, pregnancy, previous transplants).

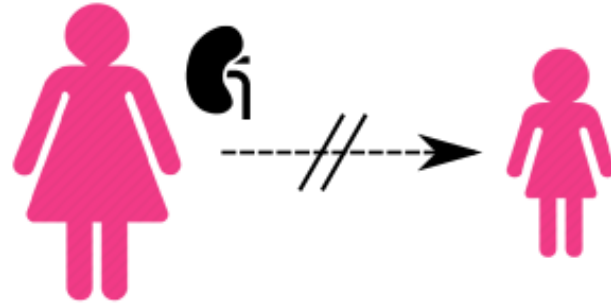
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[Rapaport, *Transpl. Proc.* 1986; Ross et al. *NEJM* 1997]



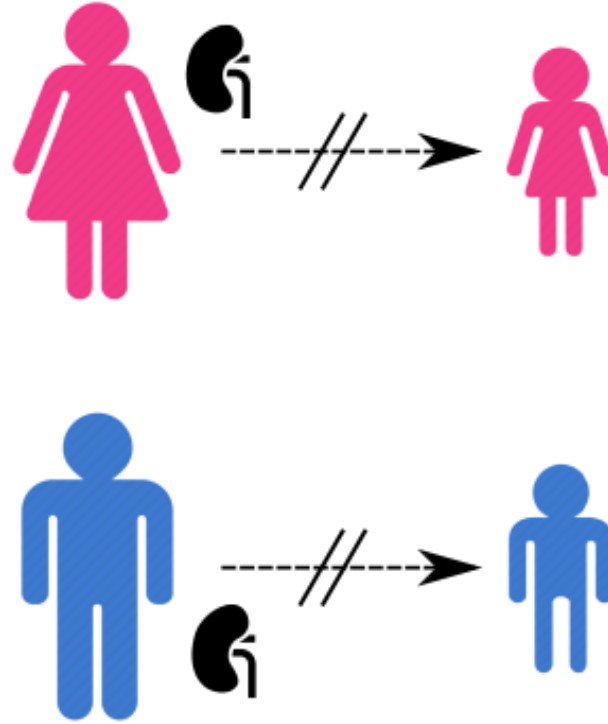
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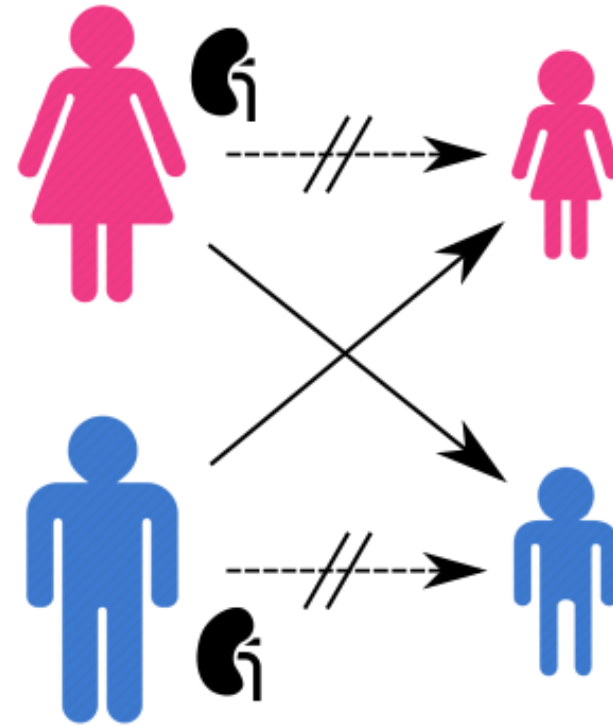
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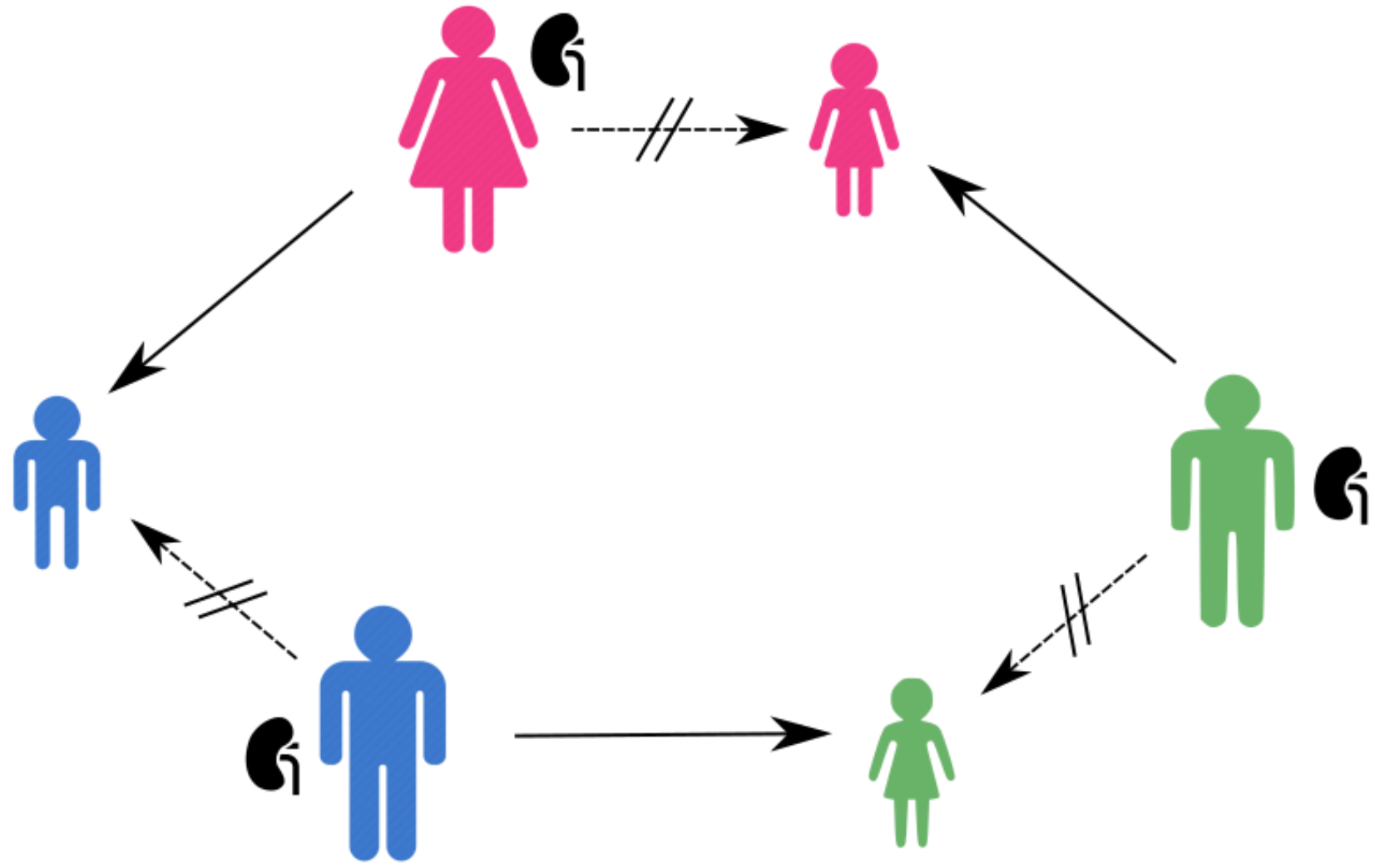


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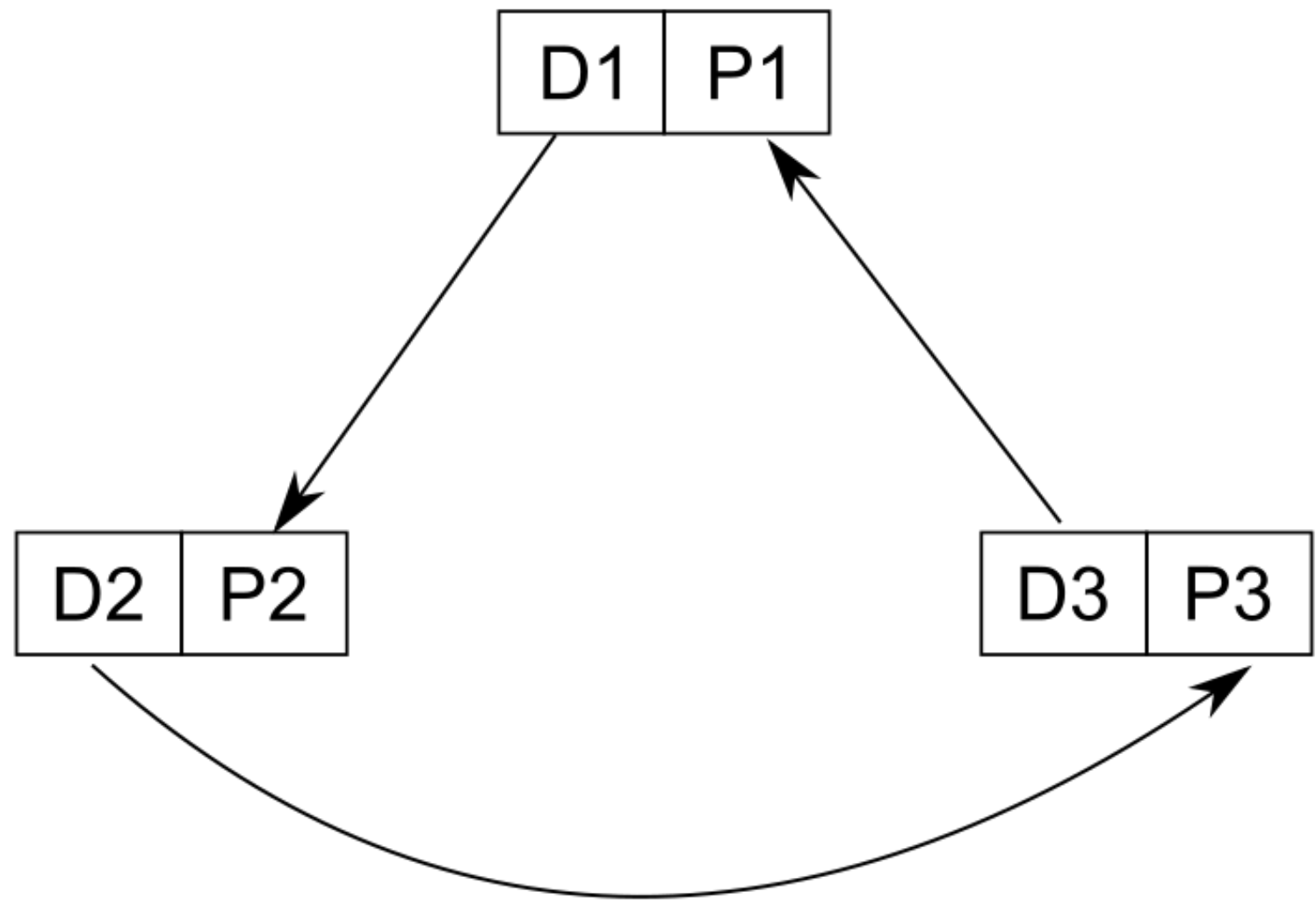
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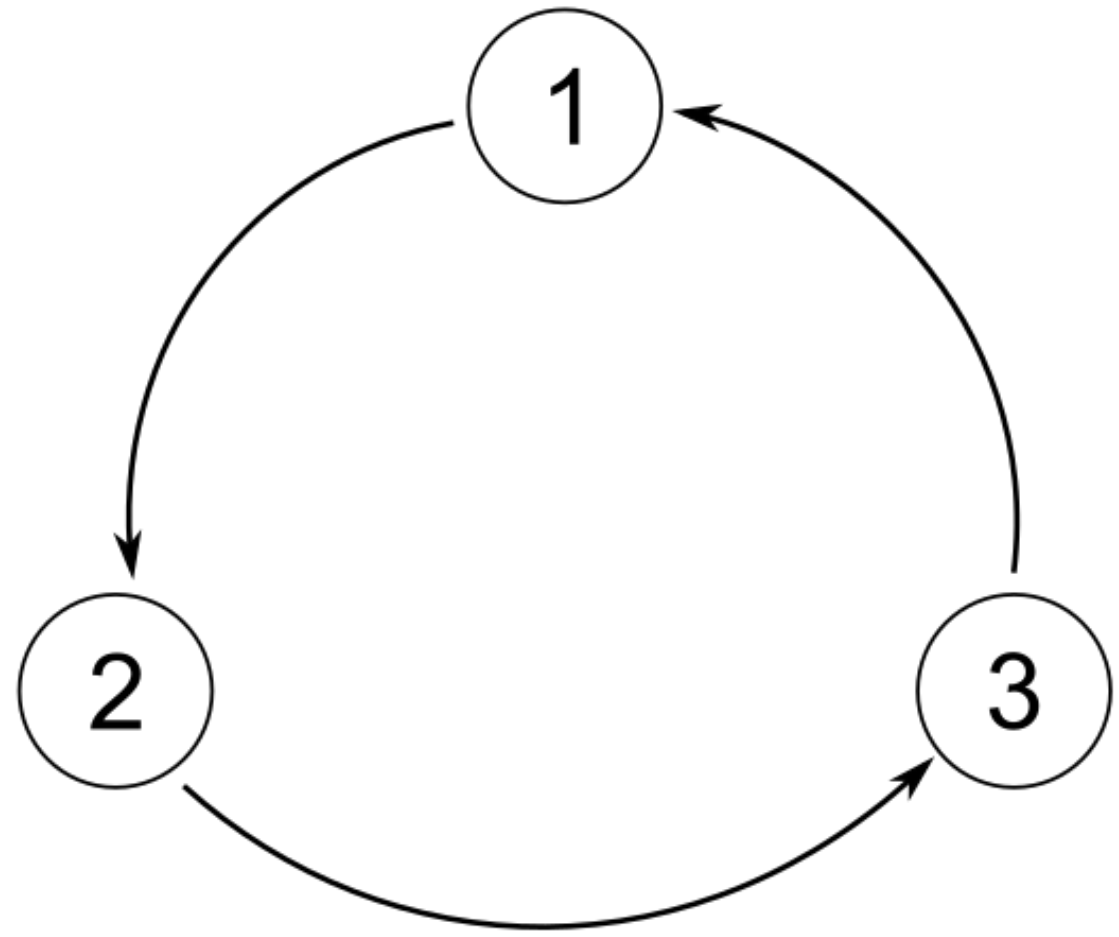
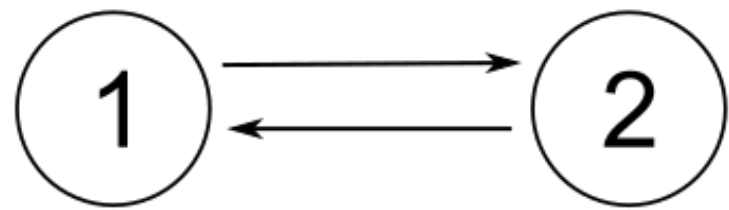
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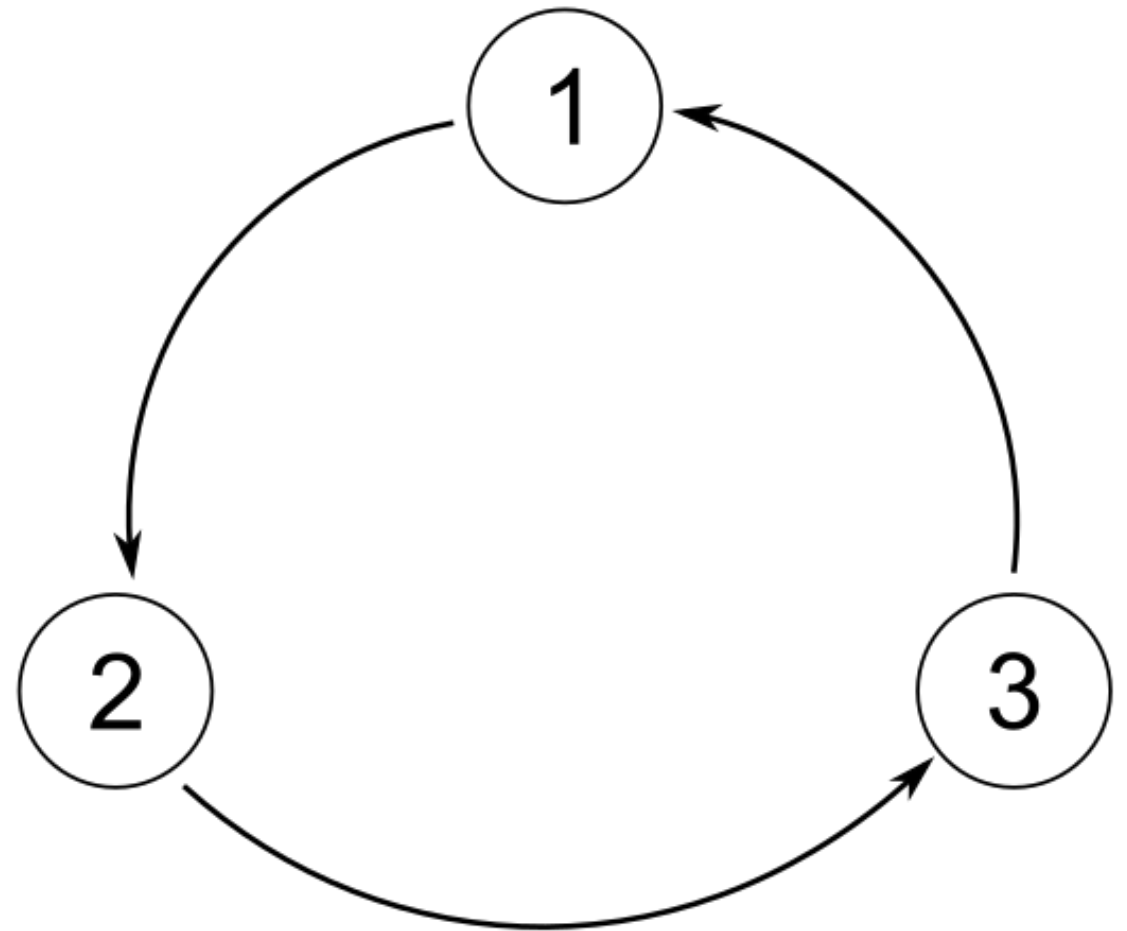
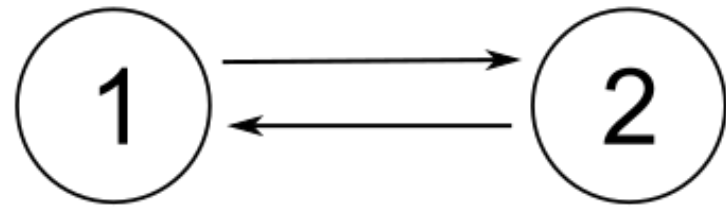
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Intuitively, having more and more of such willing-but-incompatible pairs will create more opportunities for an exchange.

## History [\[ edit \]](#)

Year	Event
1986	KPD first suggested by Rapaport <sup>[14]</sup>
1991	First KPD program started in South Korea by Dr. Park <sup>[8]</sup>
1999	First European KPD transplants performed in Switzerland <sup>[8]</sup>
2000	First KPD transplants performed in USA at the Rhode Island Hospital <sup>[8]</sup>
2001	Hopkins completes KPD transplants and begins first KPD program in USA <sup>[15]</sup>
2004	Dutch established first national KPD program <sup>[8]</sup>
2007	NEAD chain started by APD utilizing first bridge donor <sup>[16]</sup>
2007	Charlie Norwood Living Organ Donation Act clarifies legality of KPD in USA <sup>[17]</sup>
2008	National Kidney Registry organizes its first transplants on Valentine's Day <sup>[18]</sup>
2009	Hopkins leads first 16 patient multicenter Domino Chain <sup>[19][20]</sup>
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# Early Proposals for Kidney Exchange

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Proposal 1

Using TTCA



Proposal 2

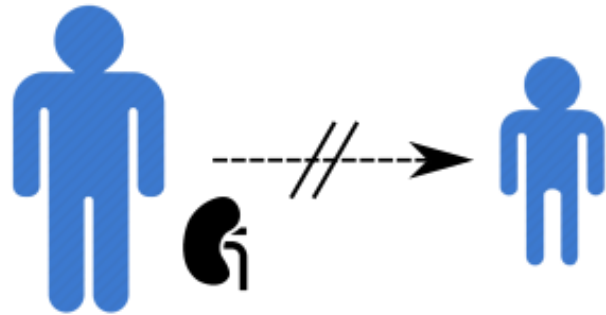
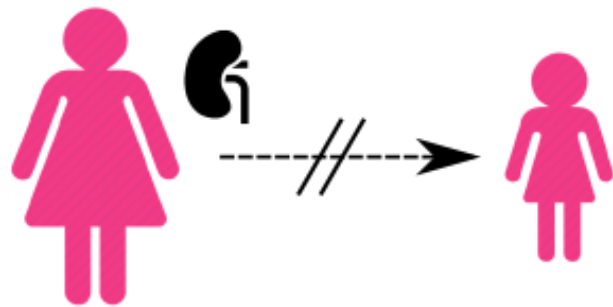


# Proposal 1: Kidney Exchange using TTCA

[Roth, Sönmez, and Ünver, *QJE* 2004]

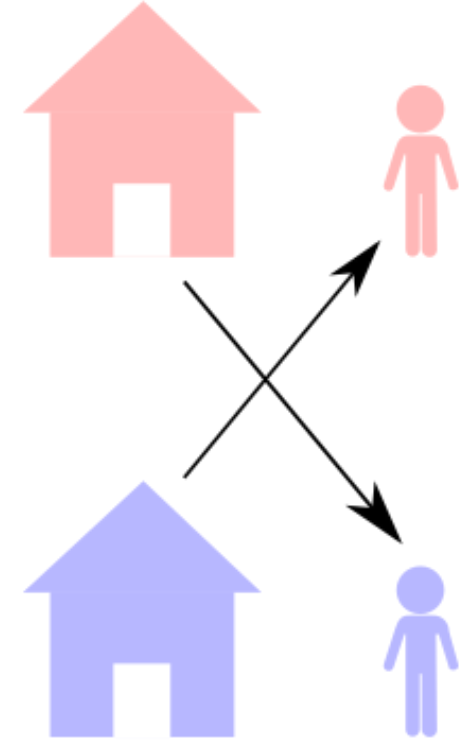
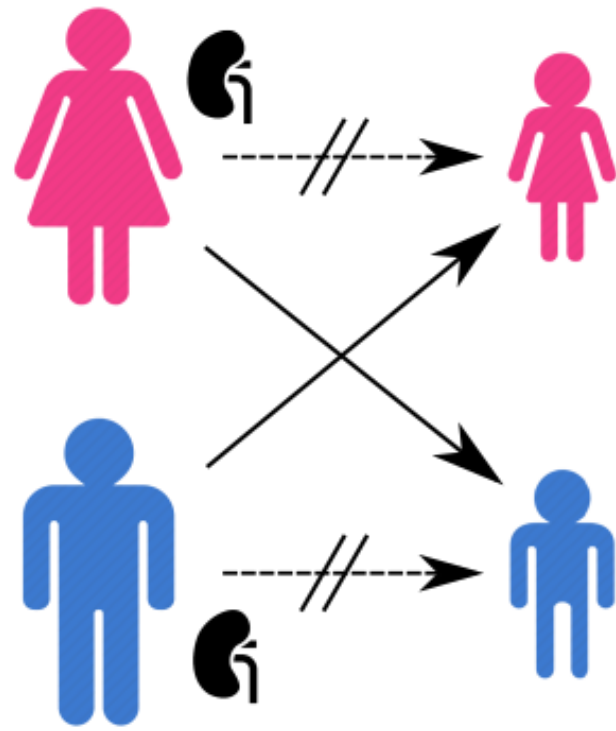
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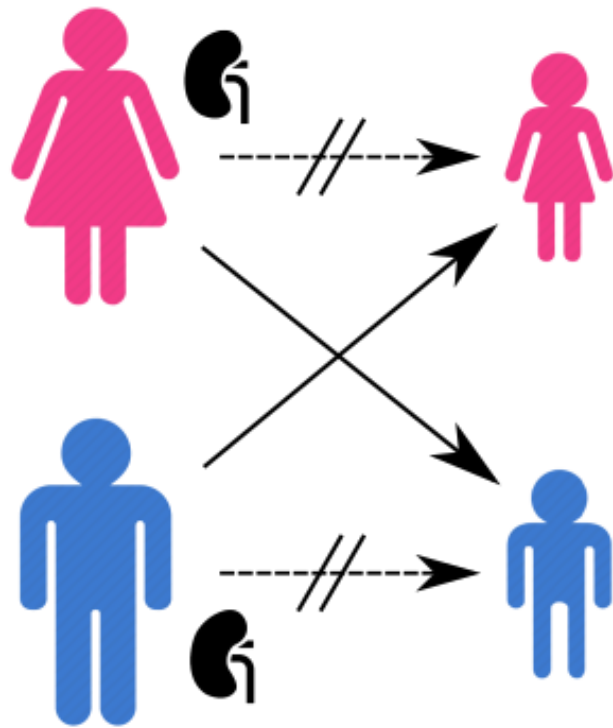
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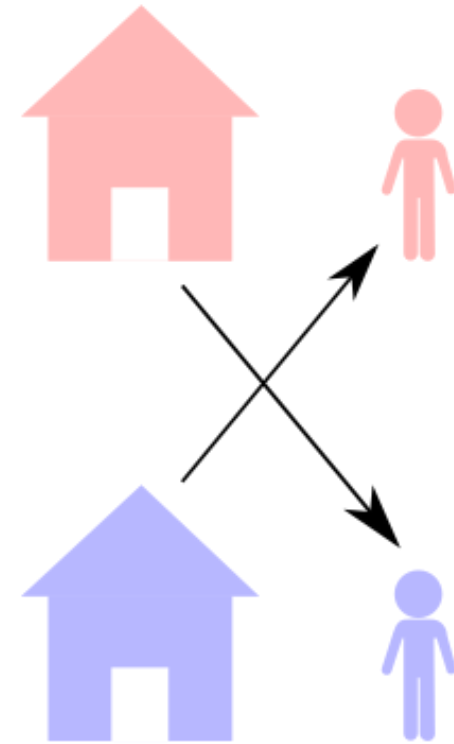


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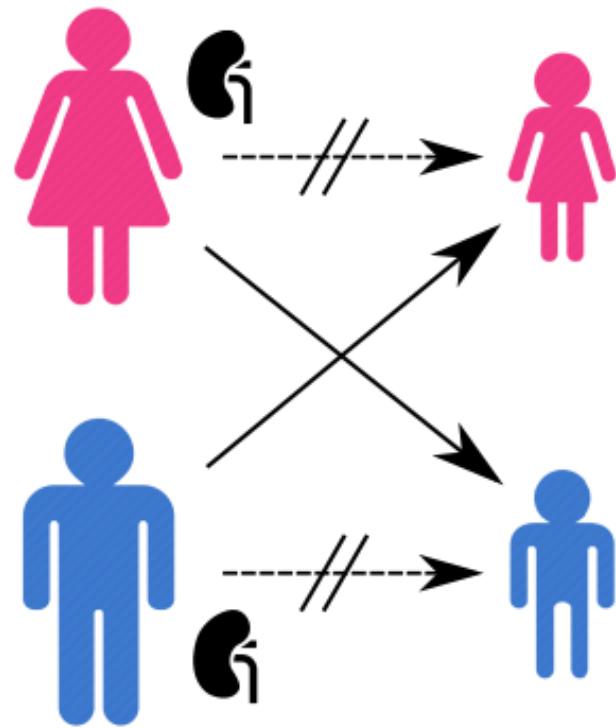
Each patient has a strict ranking of donors.



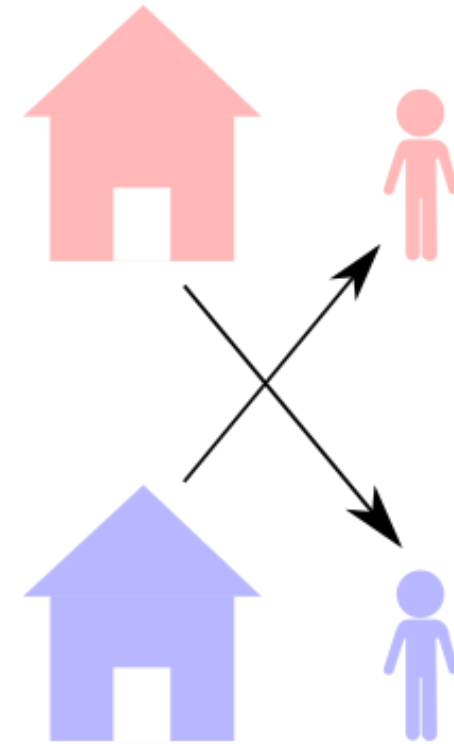
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(medically determined)



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3

4



2



1

5



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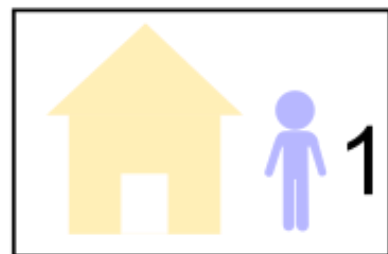
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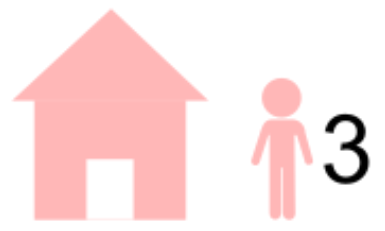
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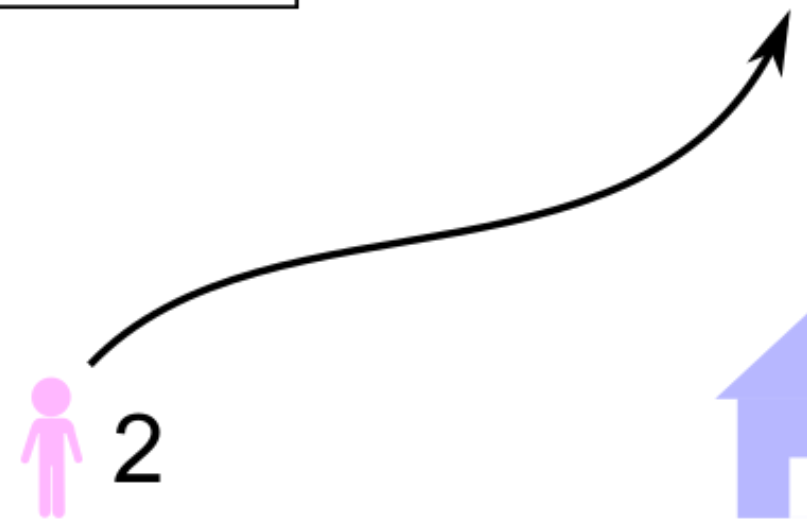
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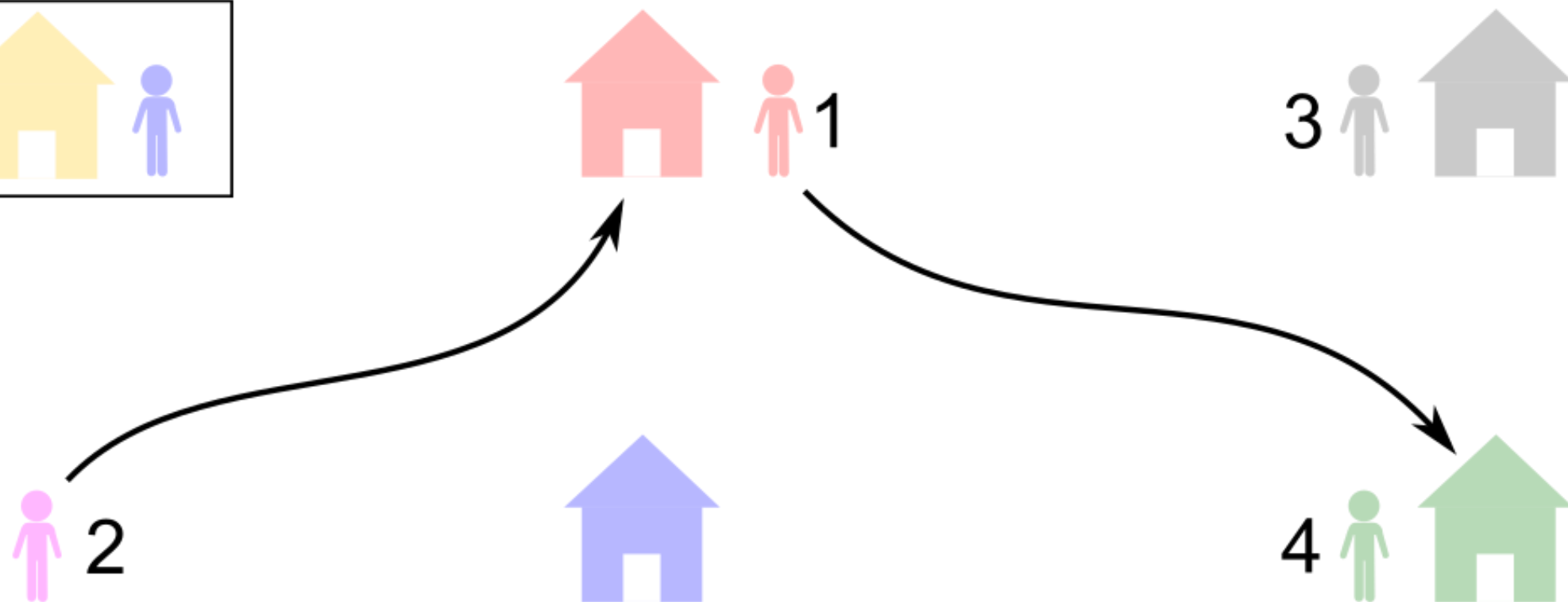
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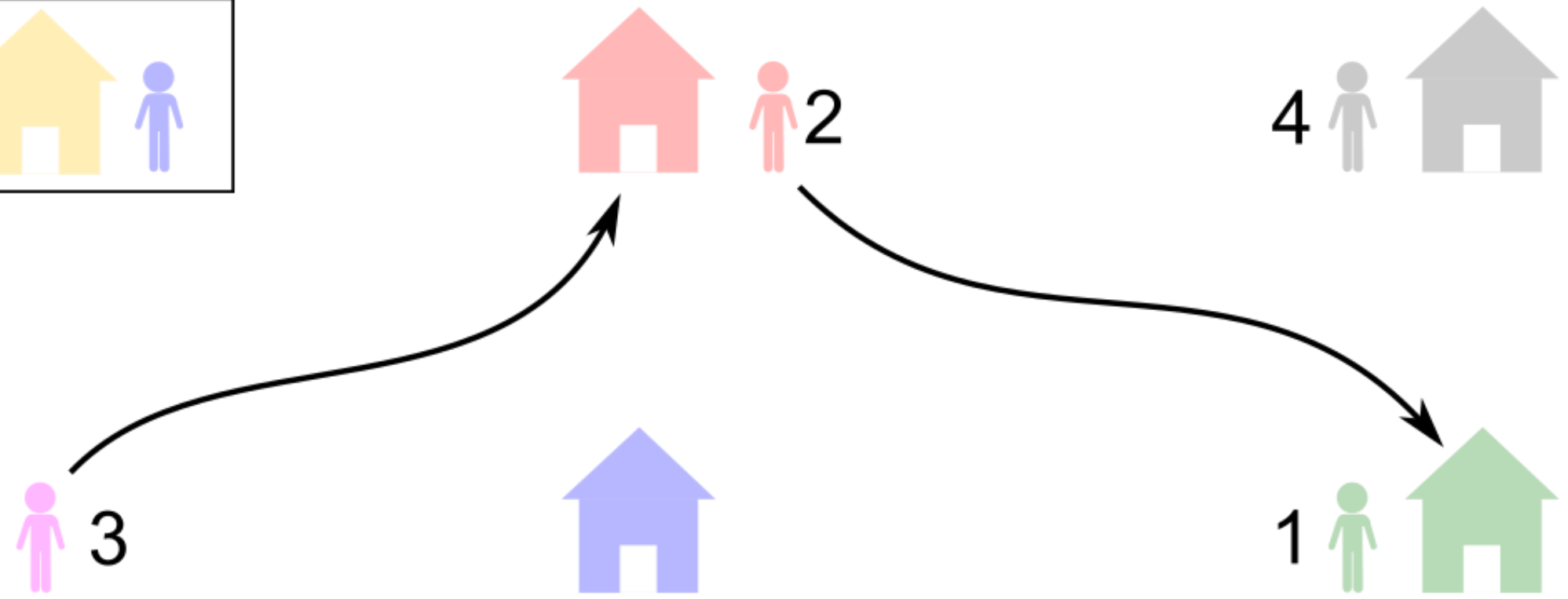
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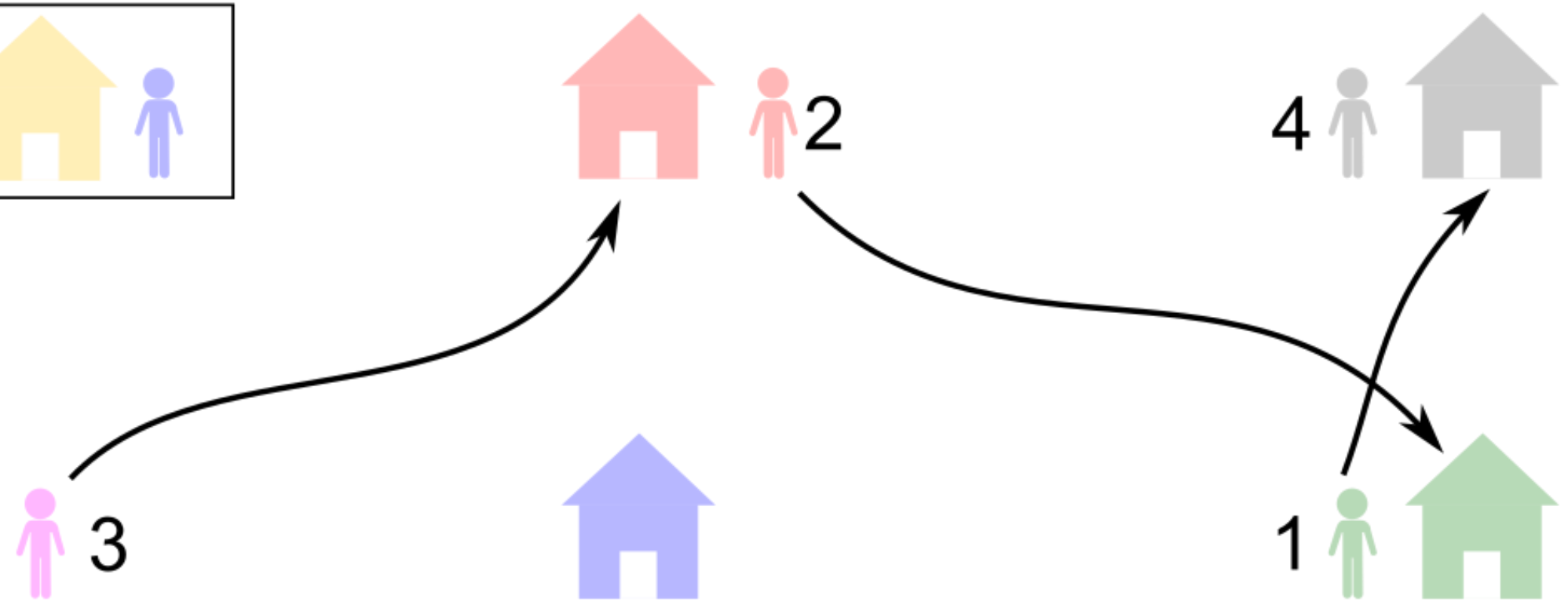
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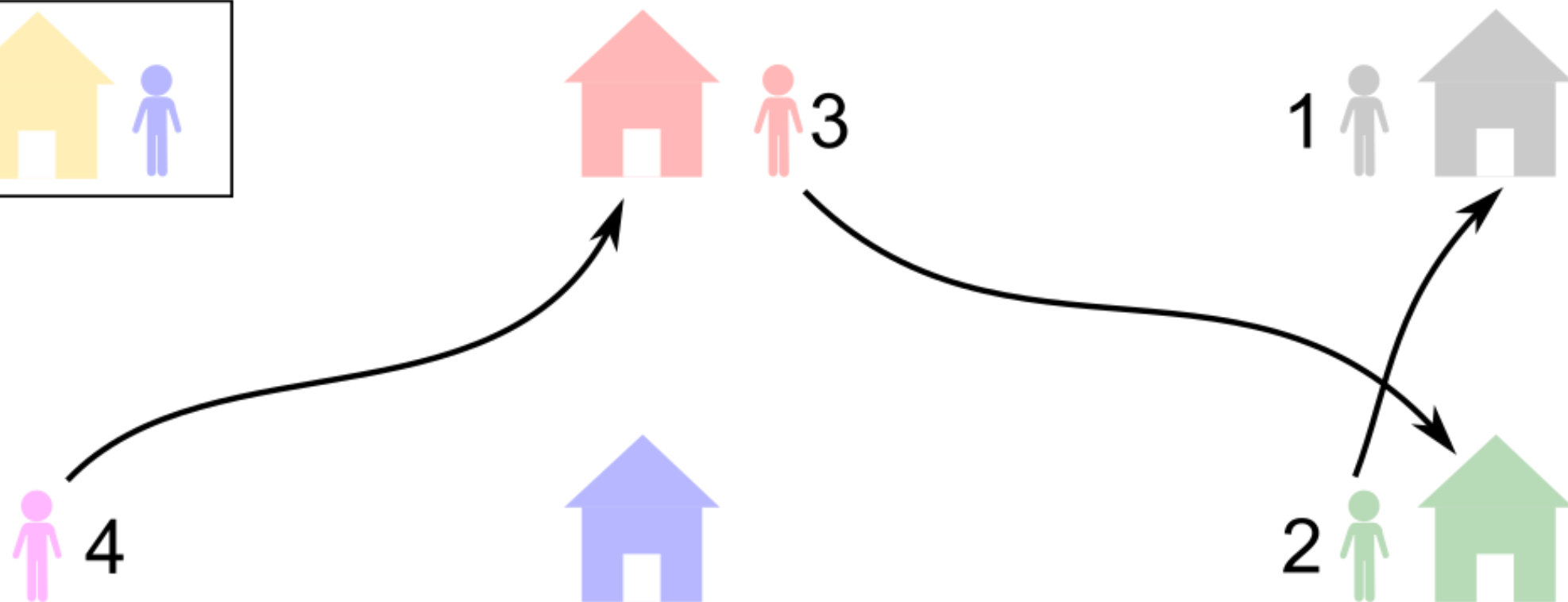
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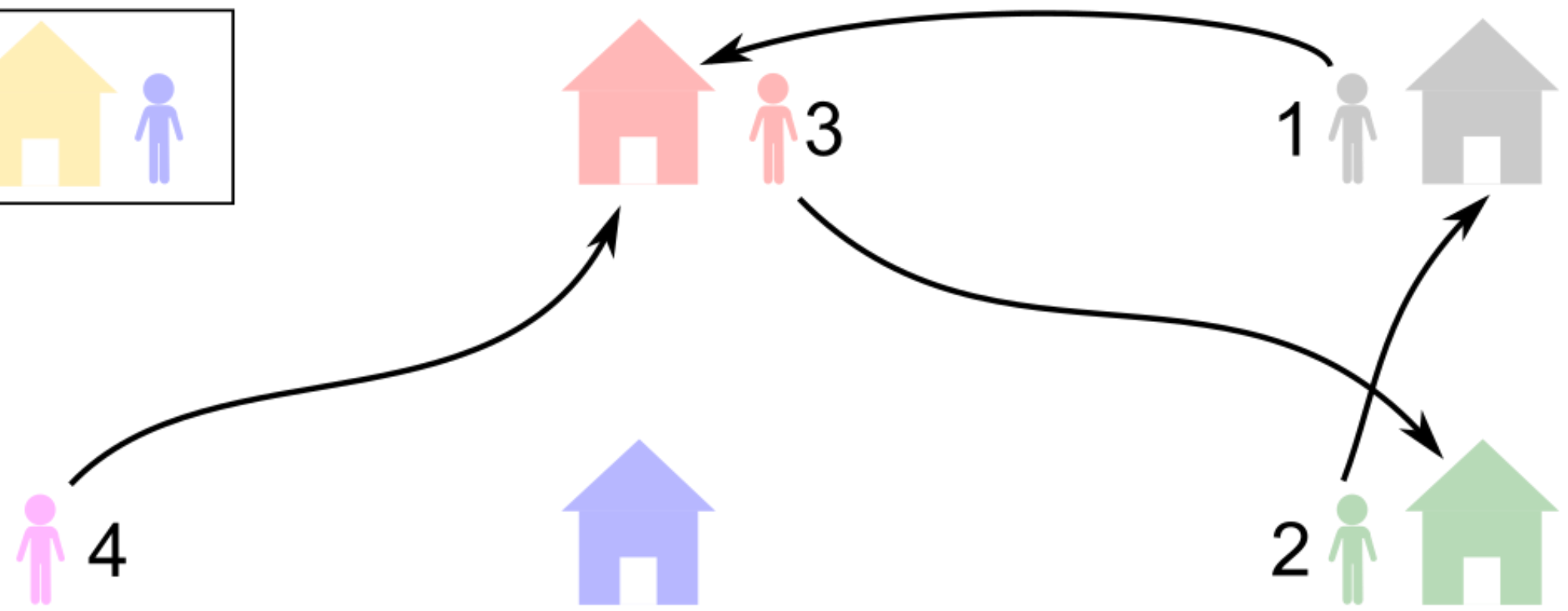
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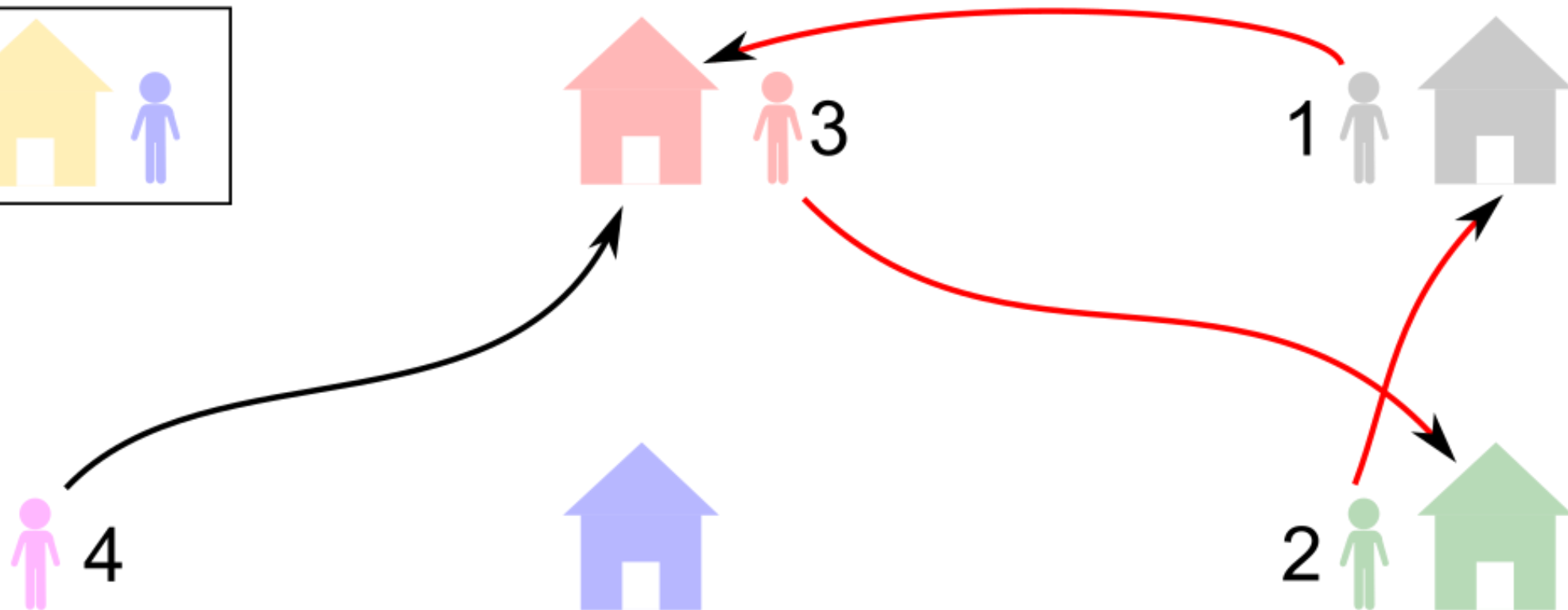
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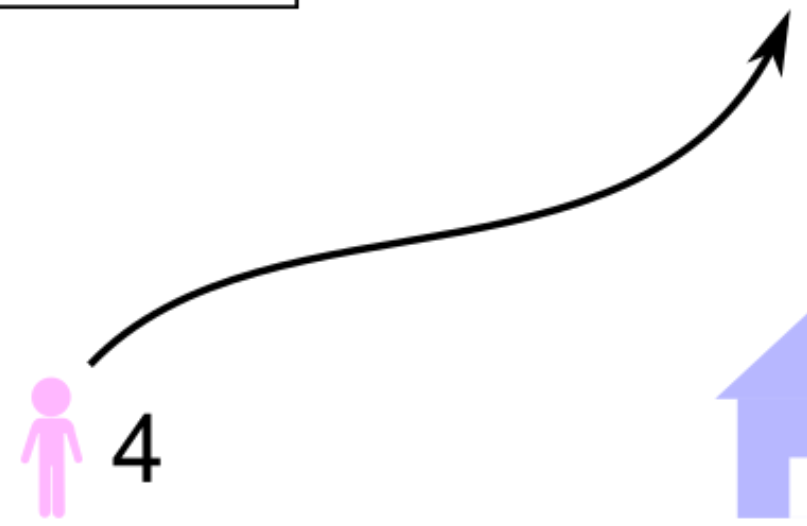
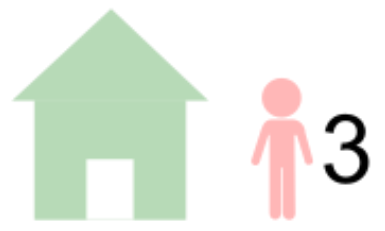
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An algorithm called "**Top trading cycles and chains**" can handle this aspect [Roth, Sönmez, and Ünver, QJE 2004].

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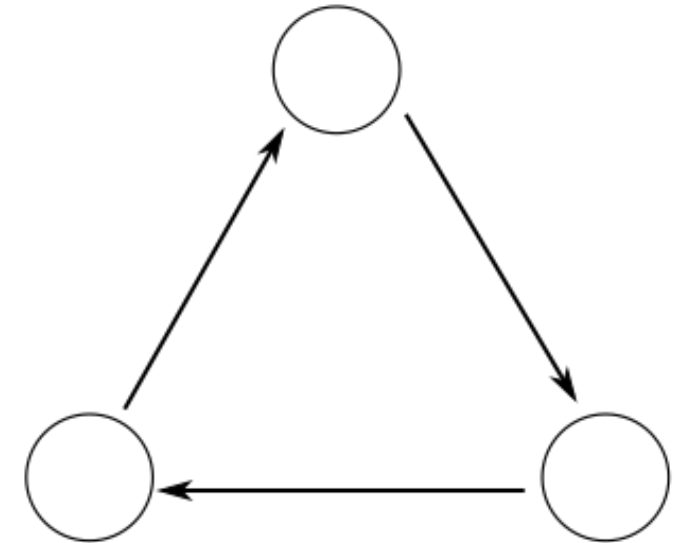
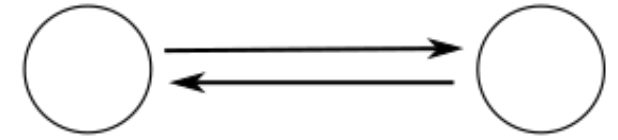
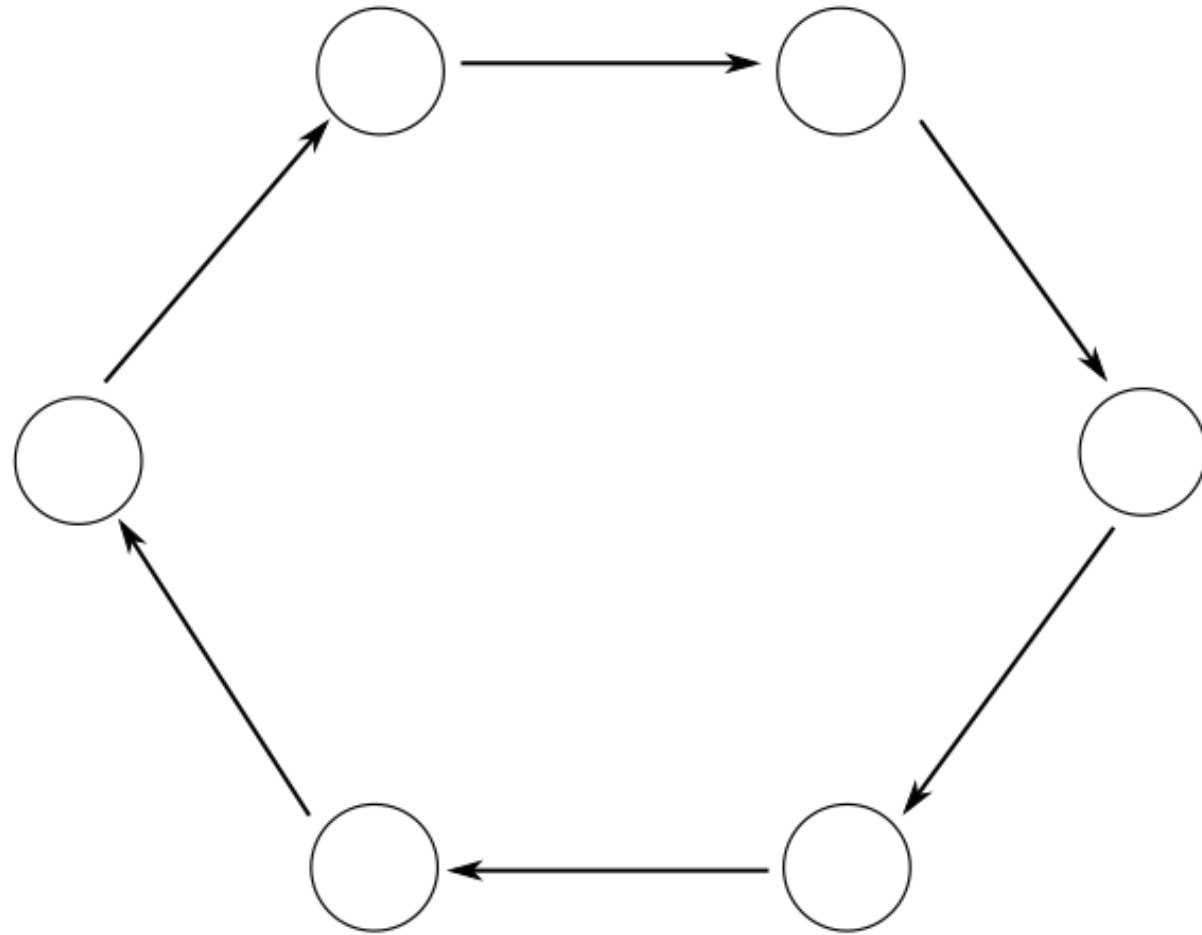


Long cycles can lead to **logistical problems**.

# Limitations of Kidney Exchange using TTCA



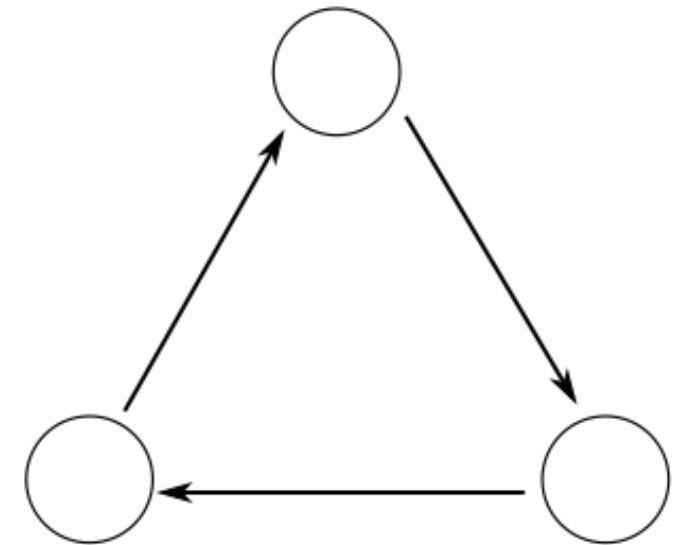
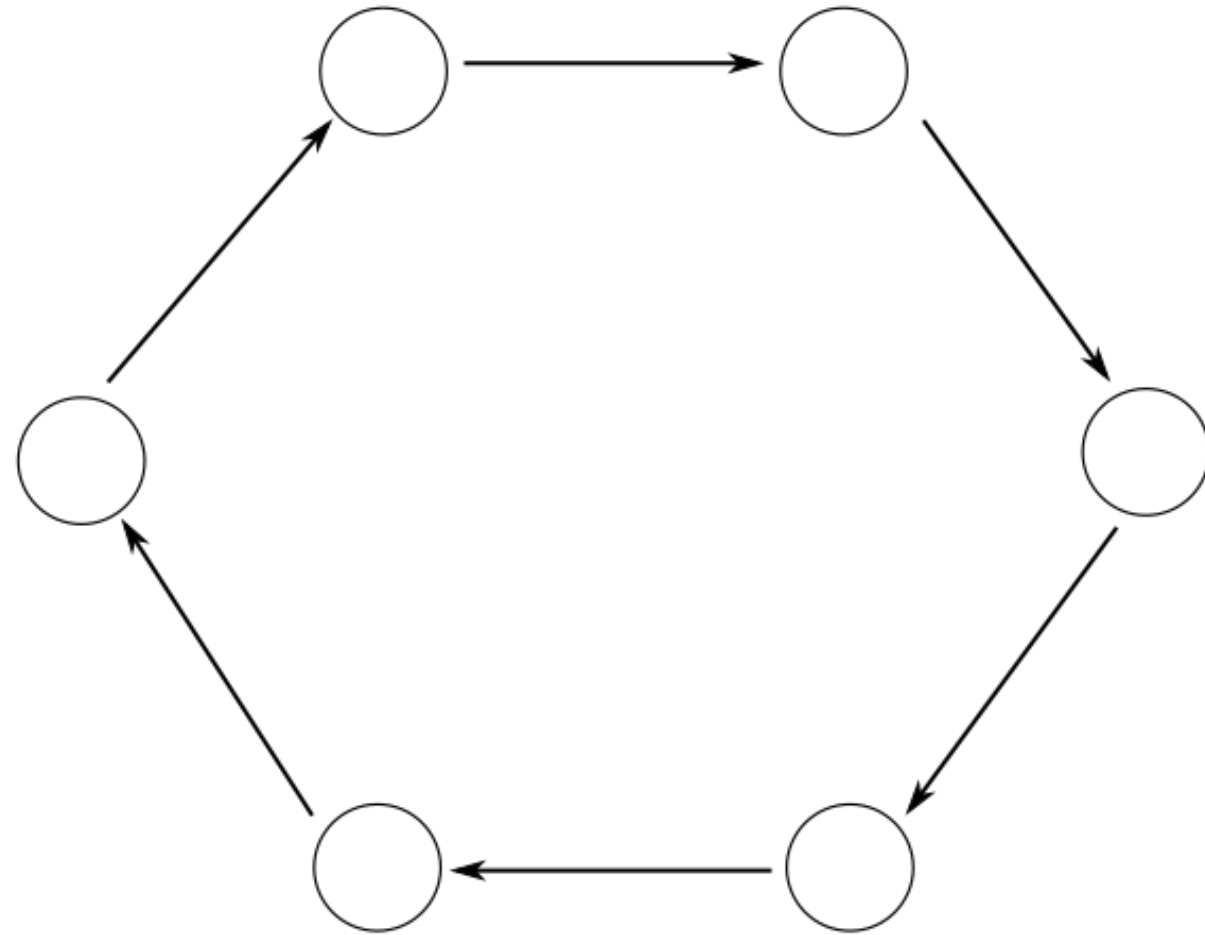
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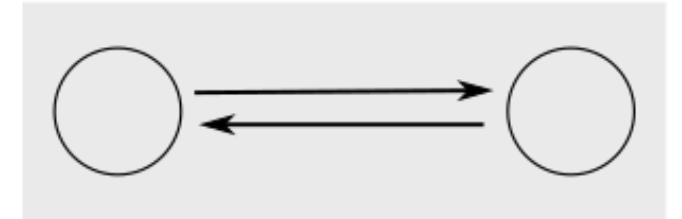
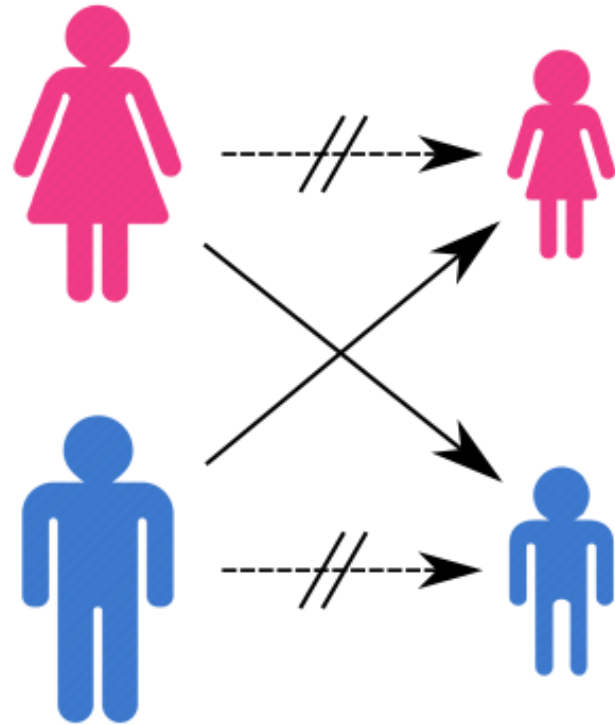




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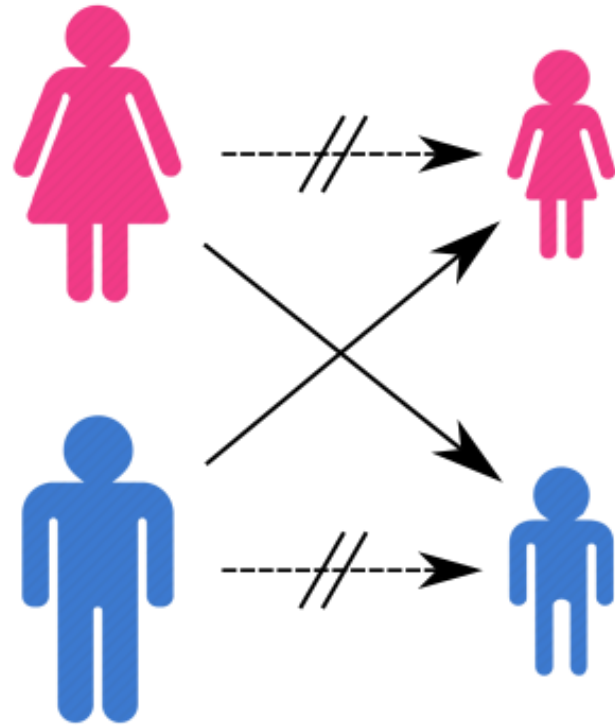
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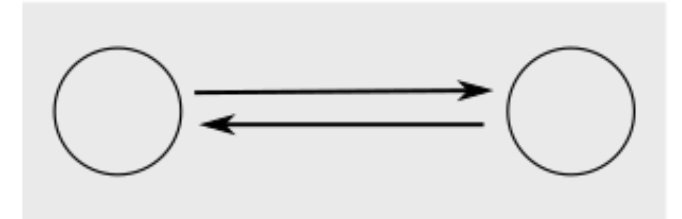
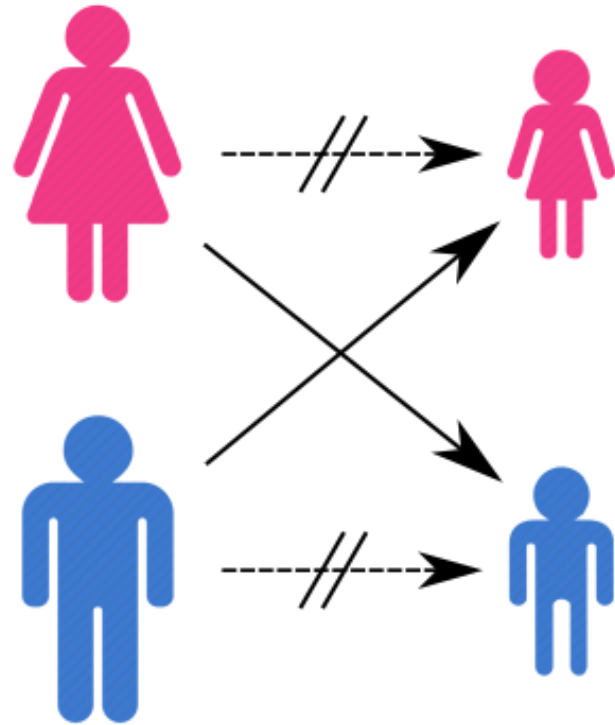


Four surgeries

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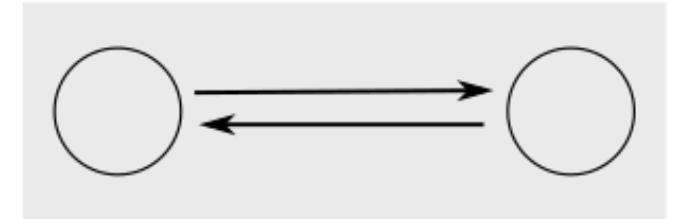
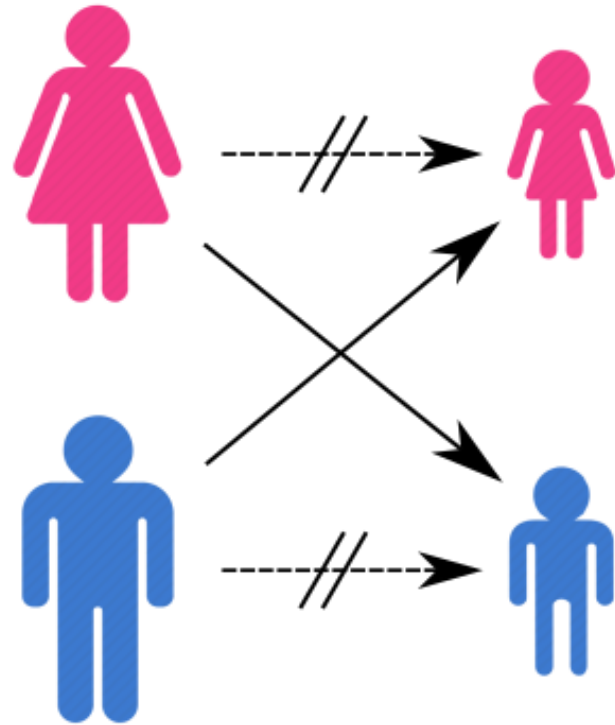


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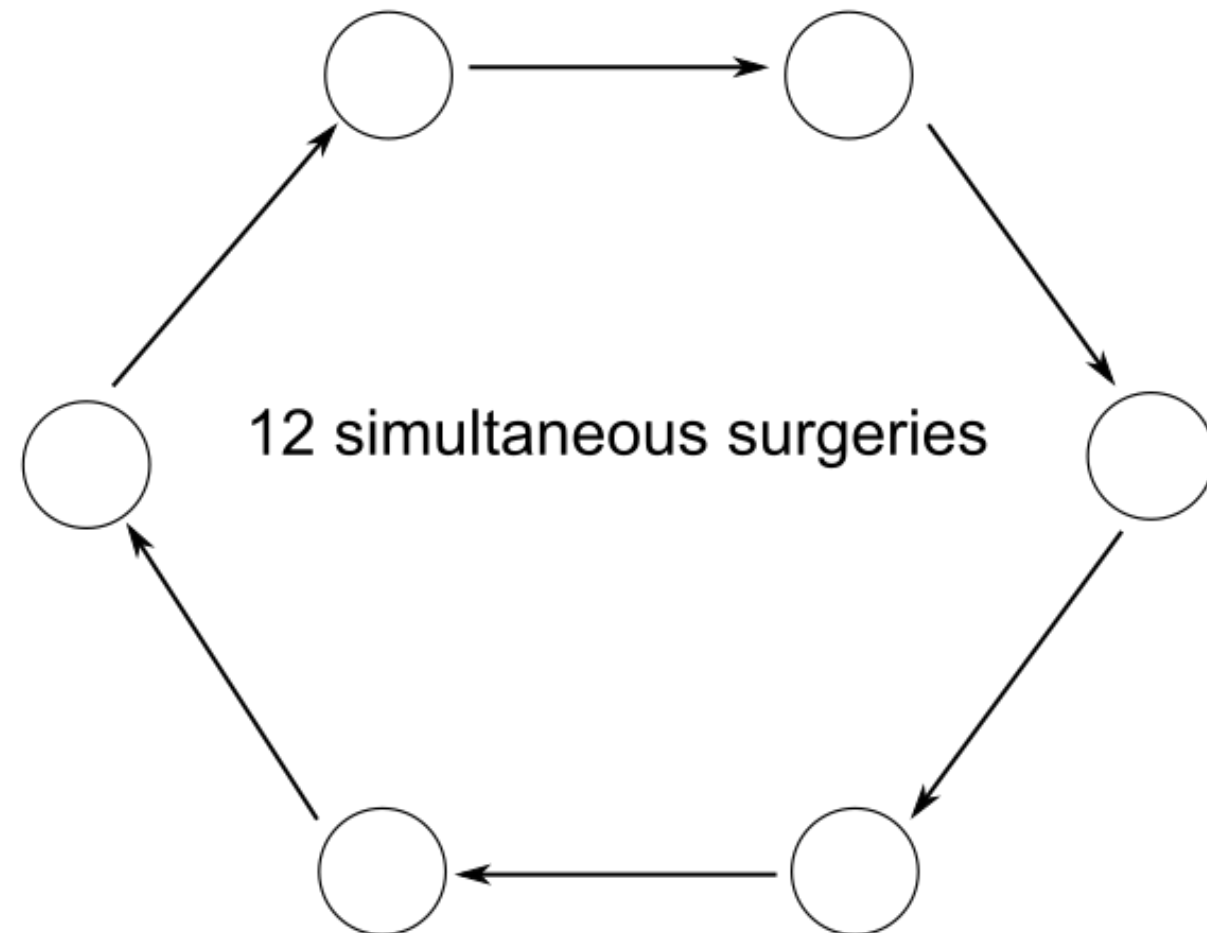
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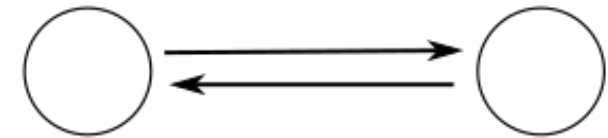
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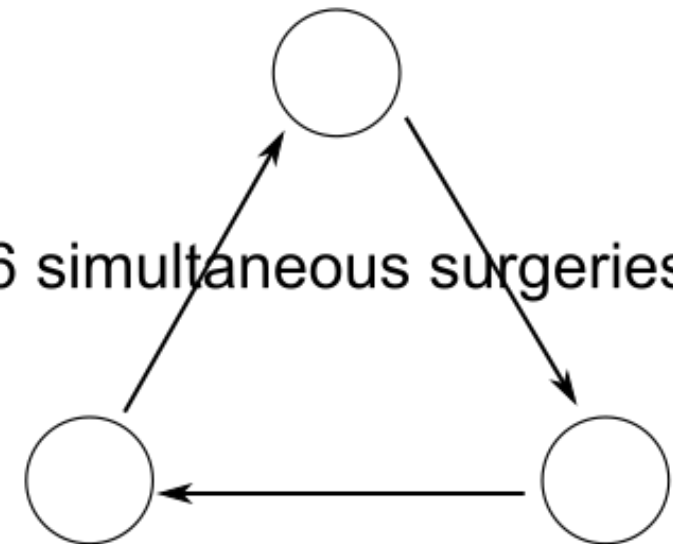
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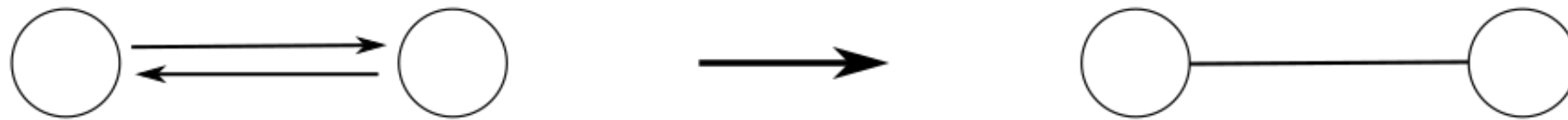
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# Early Proposals for Kidney Exchange

Proposal 1

Using TTCA



Proposal 2



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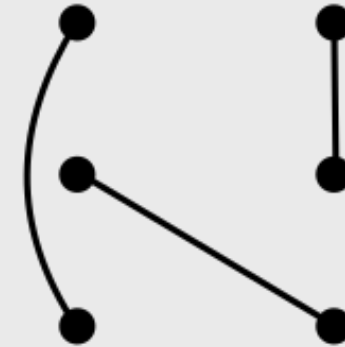
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Proposal 2

Using Matchings

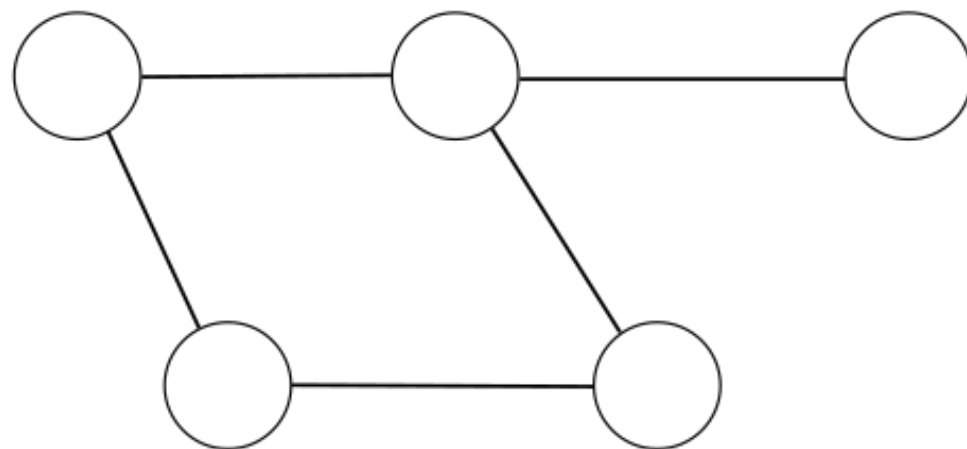


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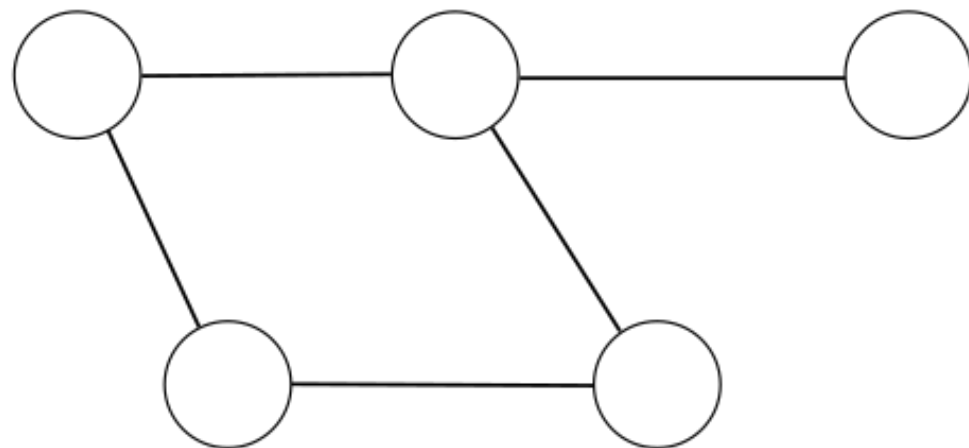
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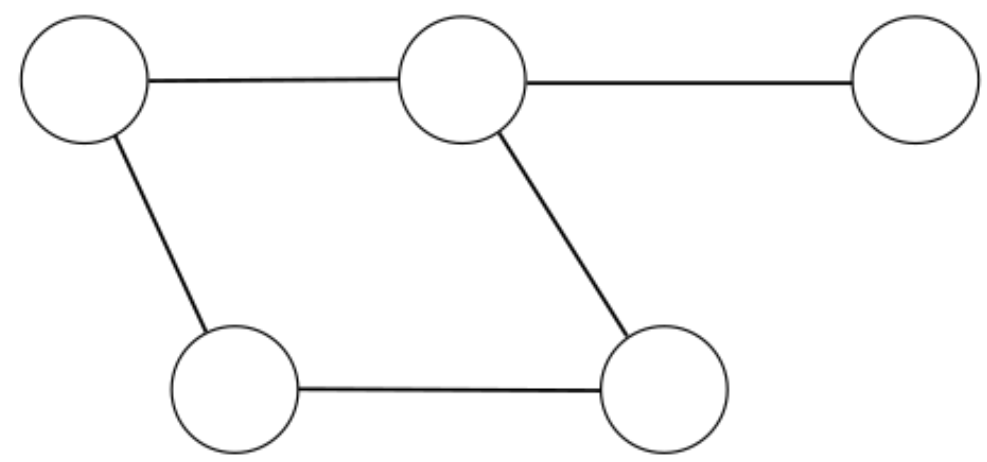
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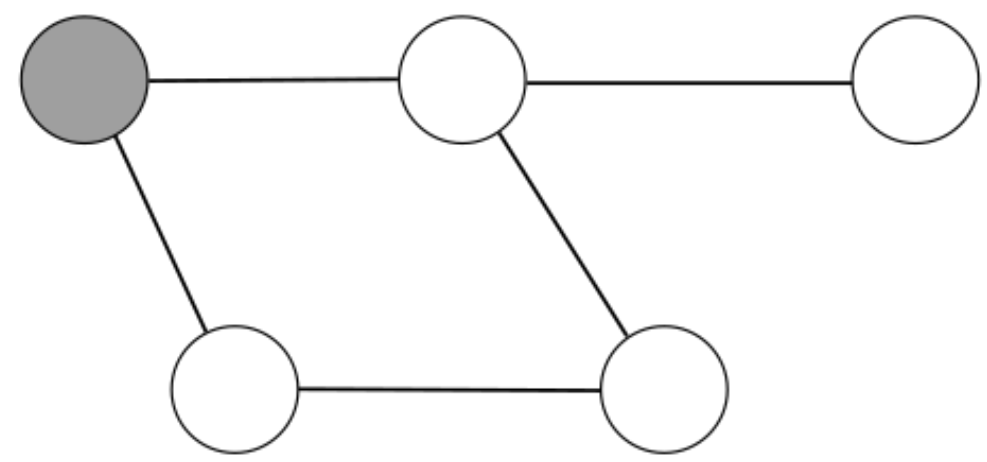


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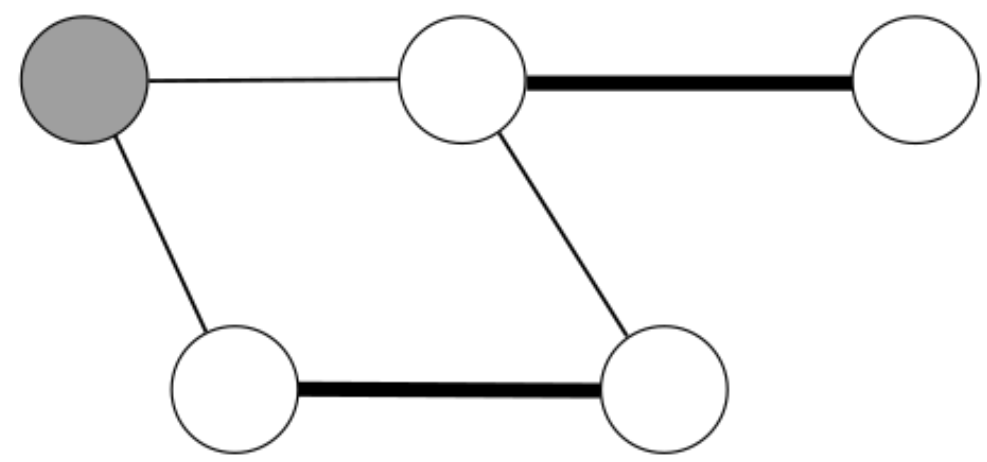


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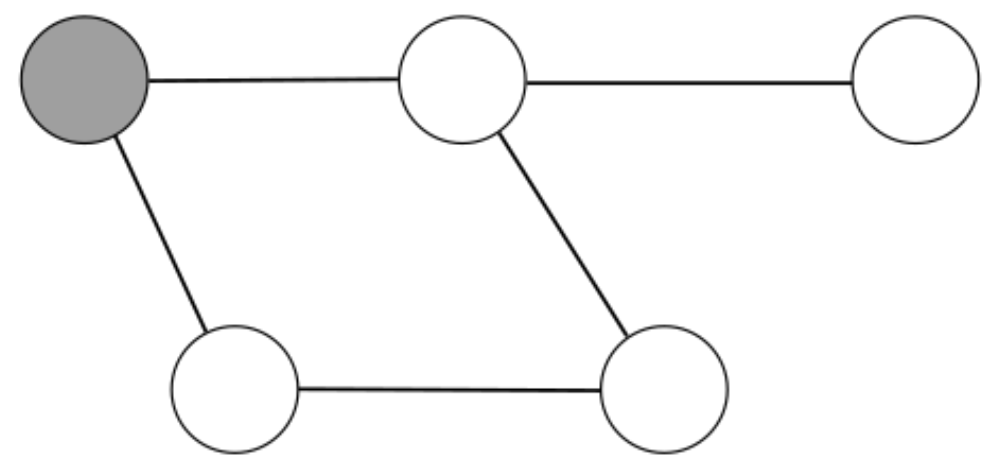
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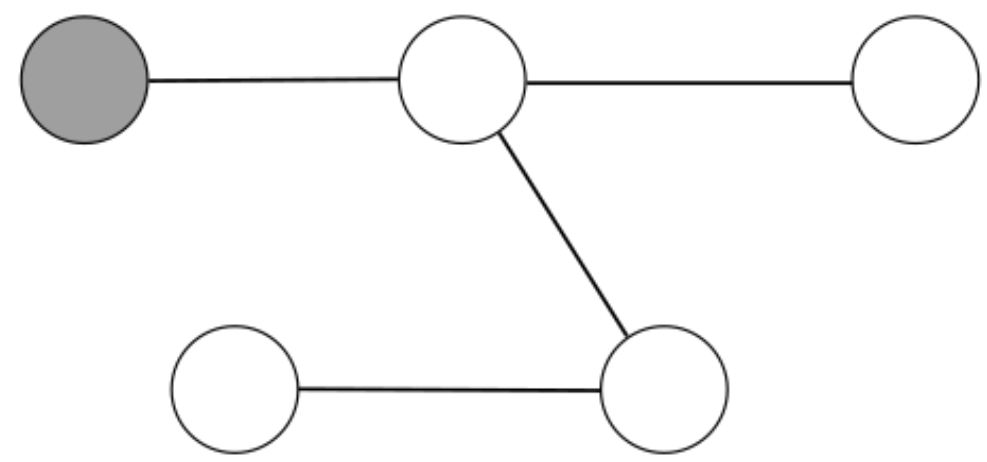


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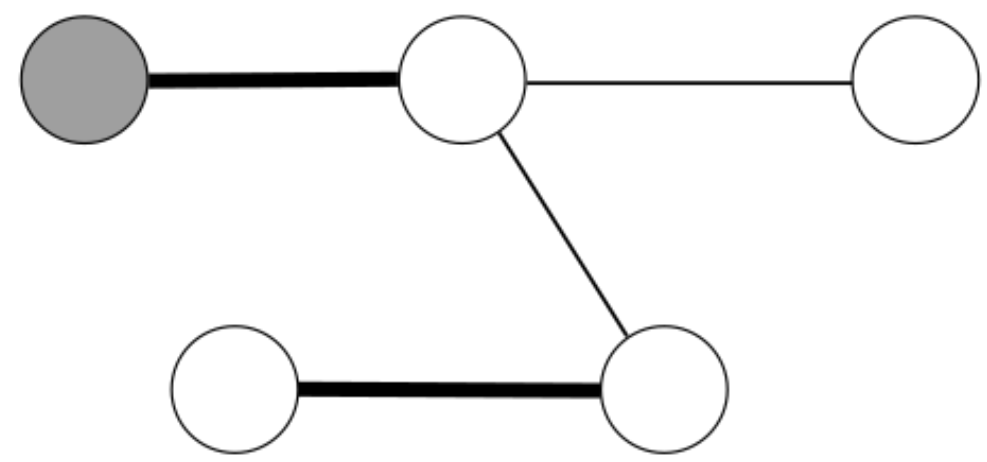


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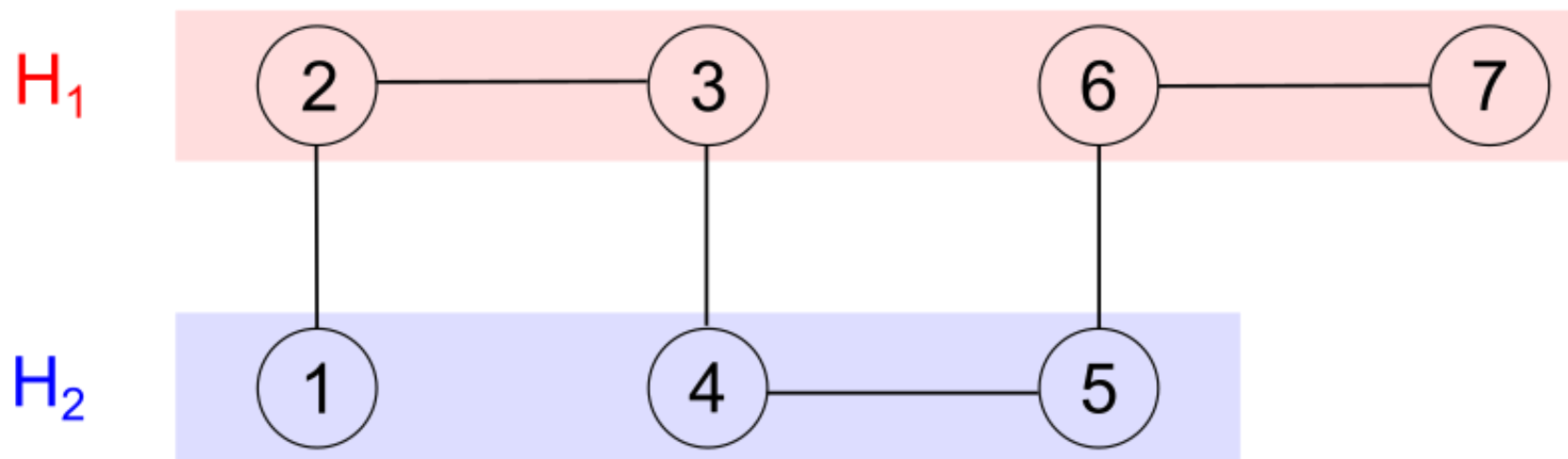
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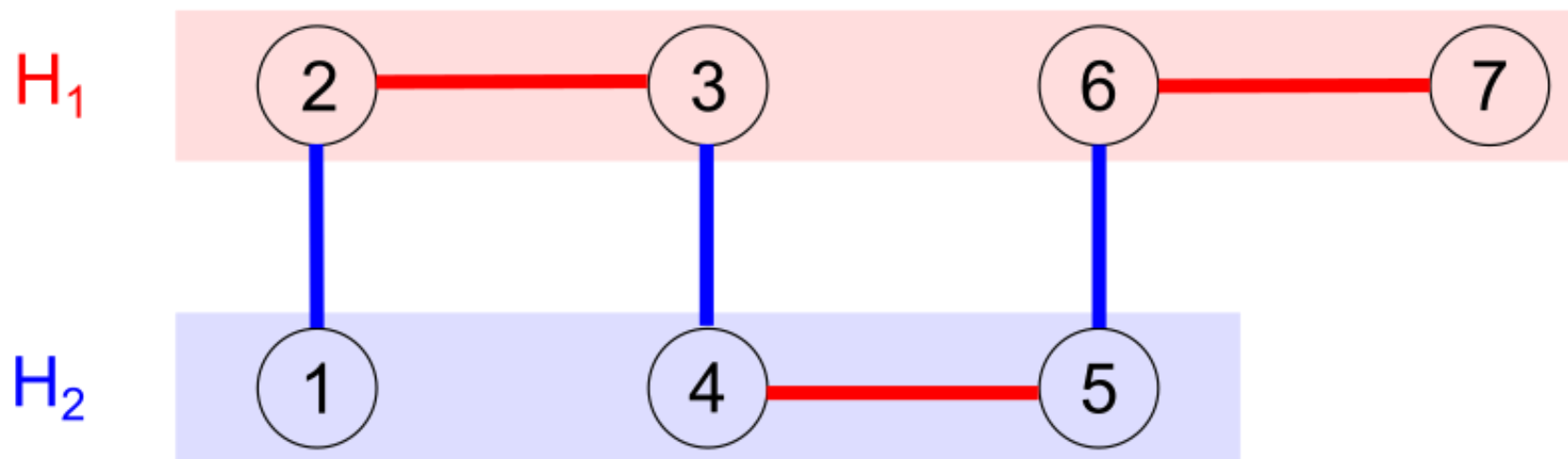
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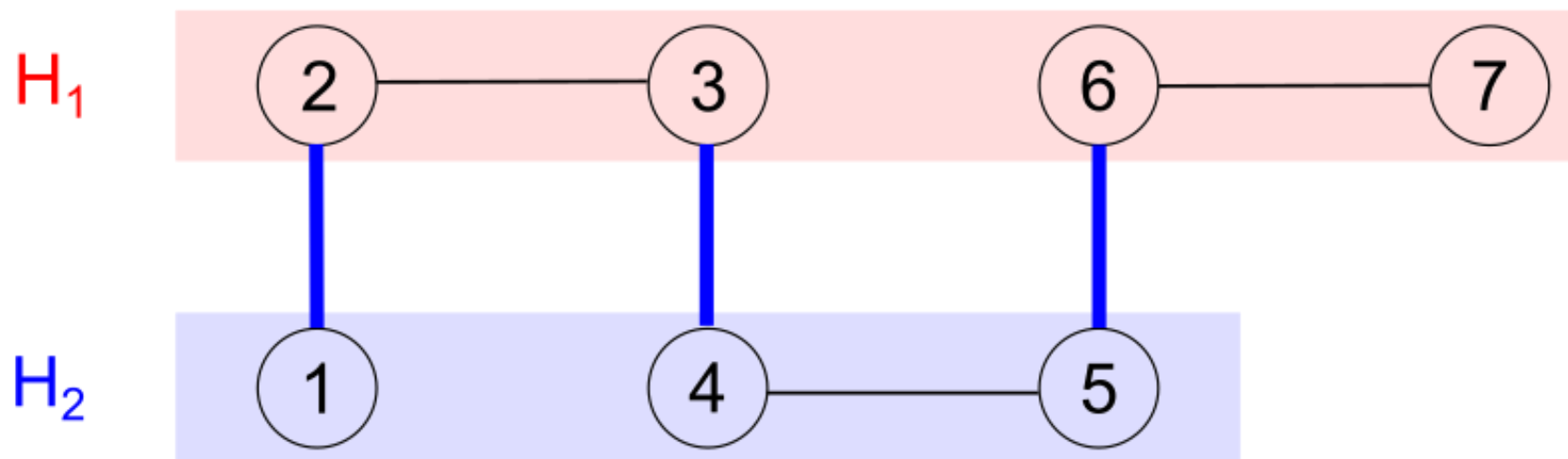
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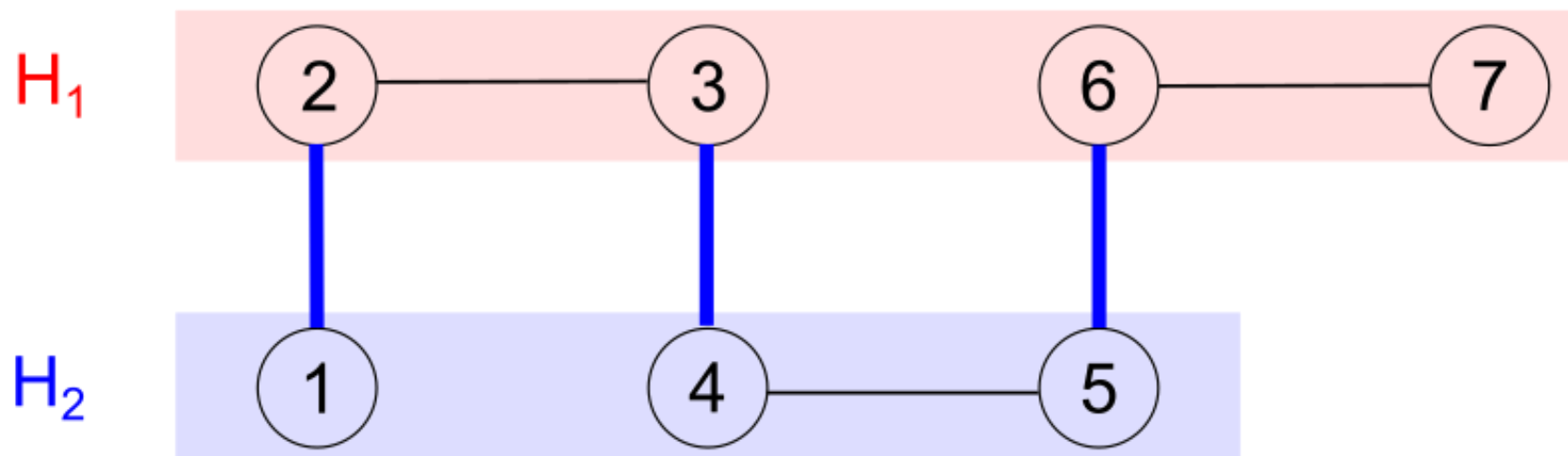
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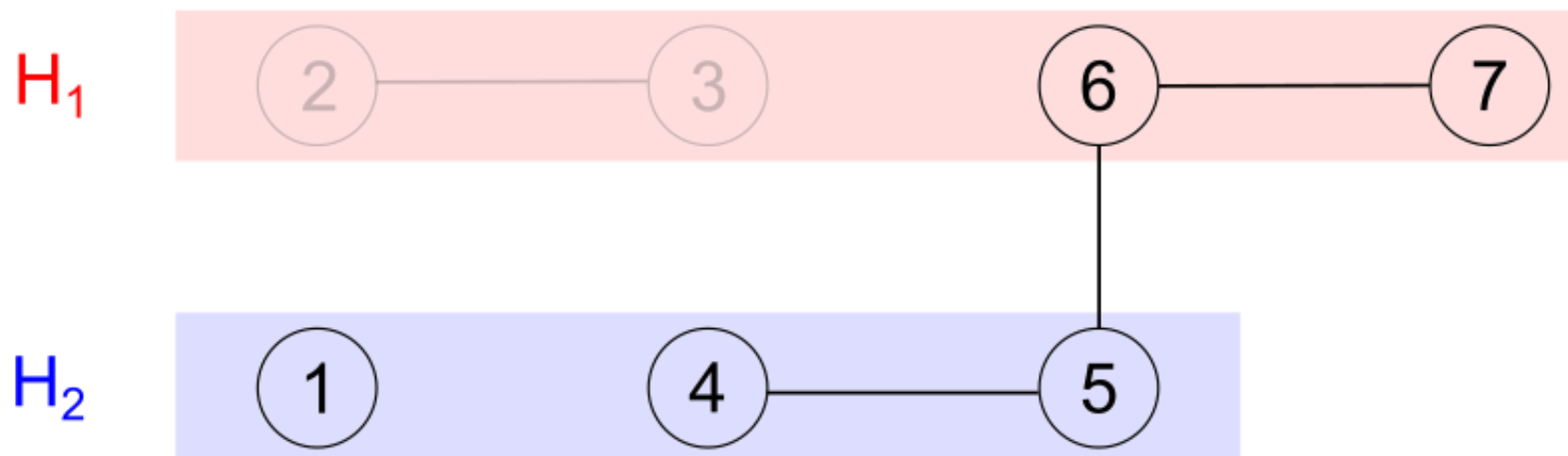


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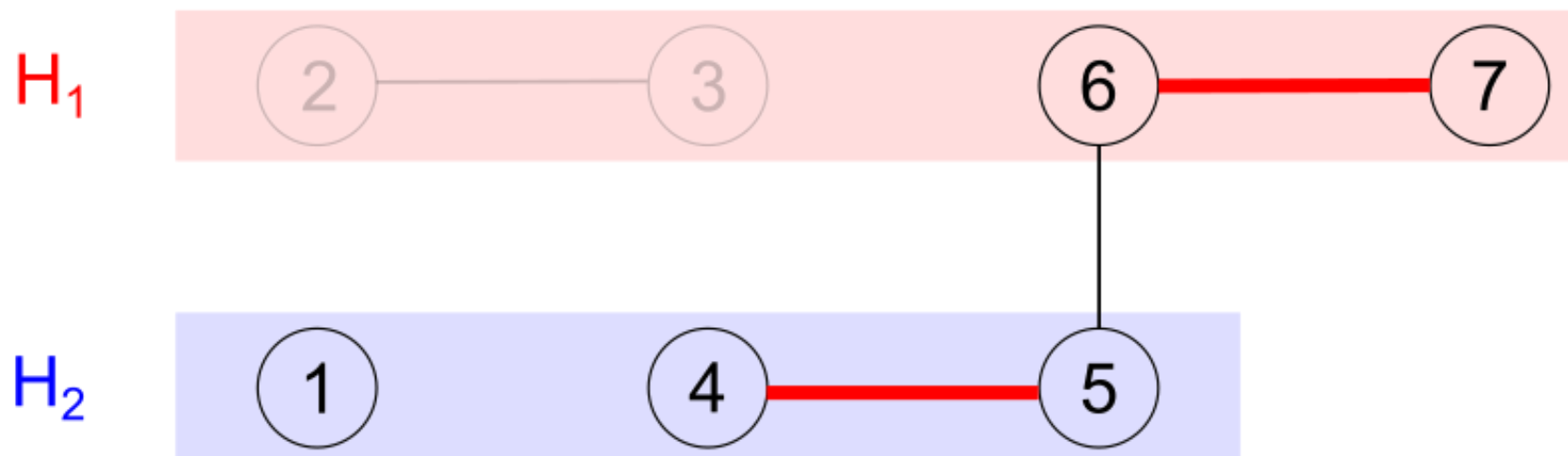


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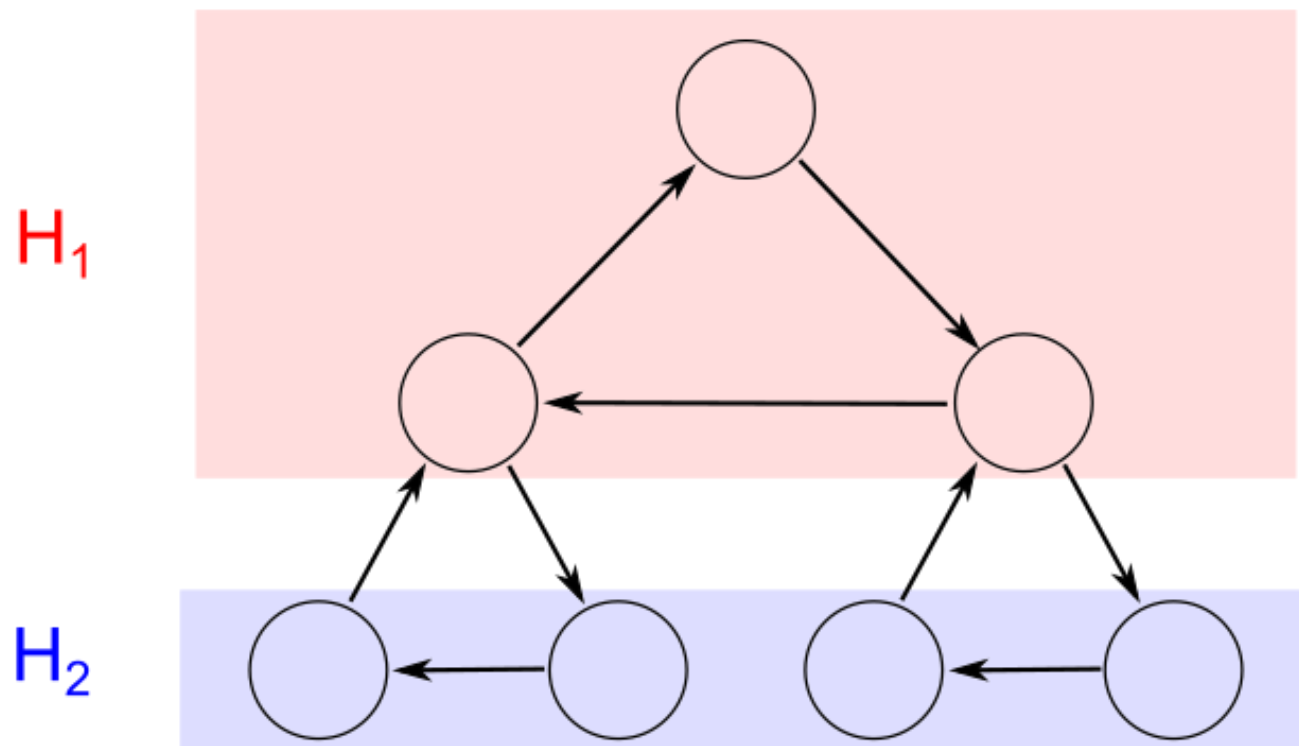
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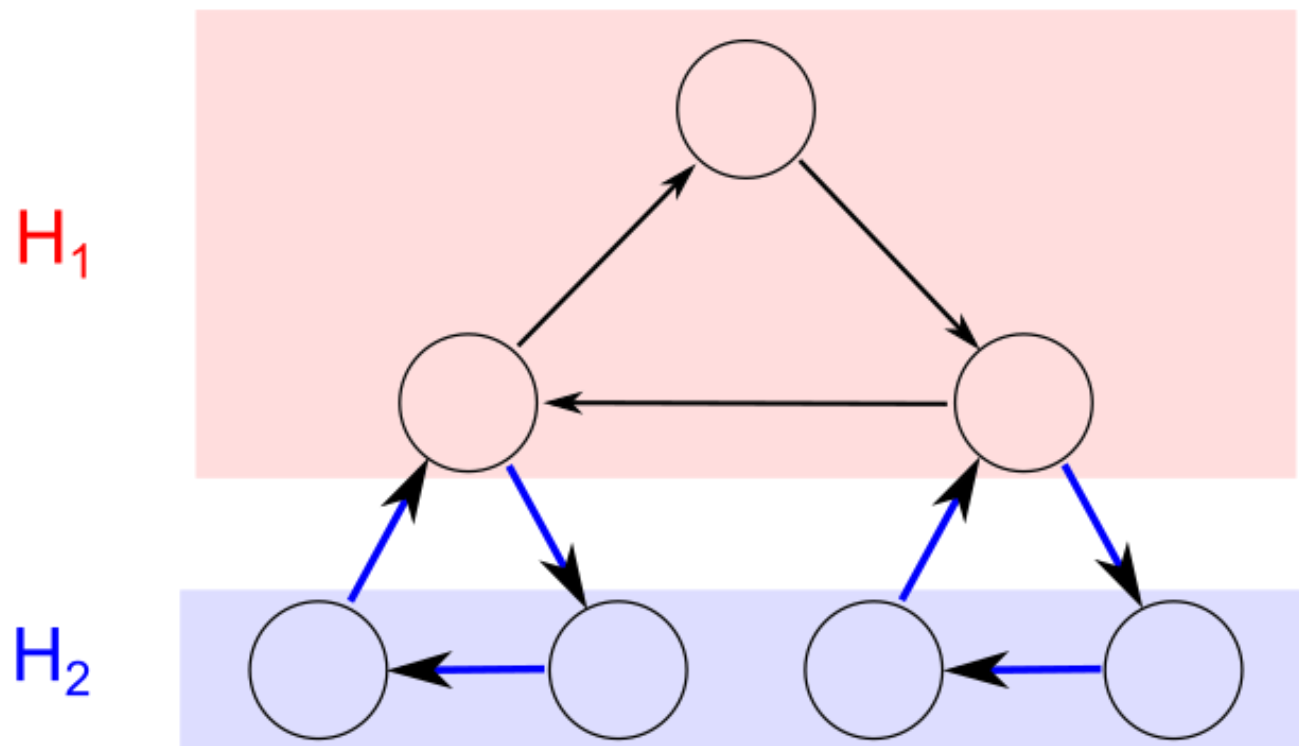
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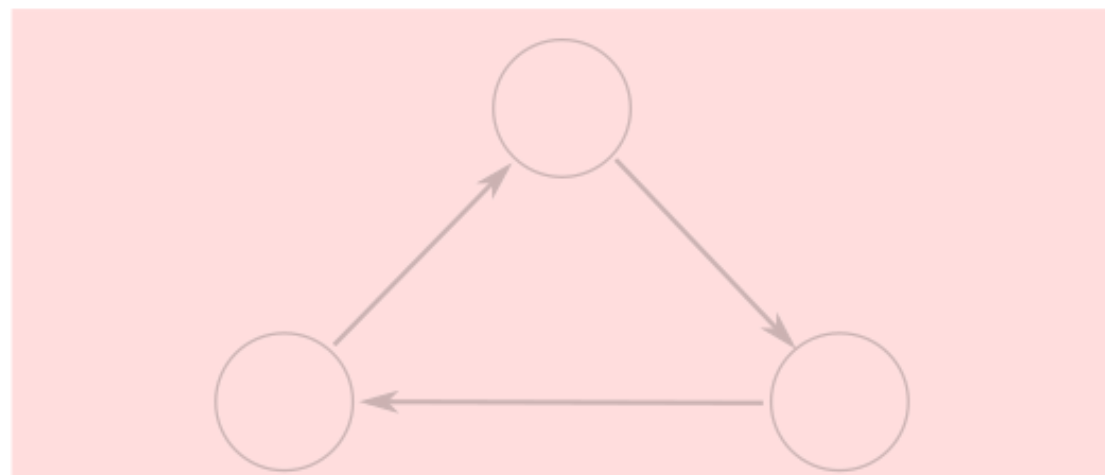


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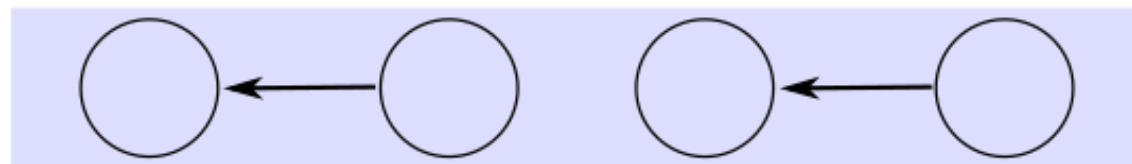


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$H_2$



## History [\[ edit \]](#)

Year	Event
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1991	First KPD program started in South Korea by Dr. Park <sup>[8]</sup>
1999	First European KPD transplants performed in Switzerland <sup>[8]</sup>
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2004	Dutch established first national KPD program <sup>[8]</sup>
2007	NEAD chain started by APD utilizing first bridge donor <sup>[16]</sup>
2007	Charlie Norwood Living Organ Donation Act clarifies legality of KPD in USA <sup>[17]</sup>
2008	National Kidney Registry organizes its first transplants on Valentine's Day <sup>[18]</sup>
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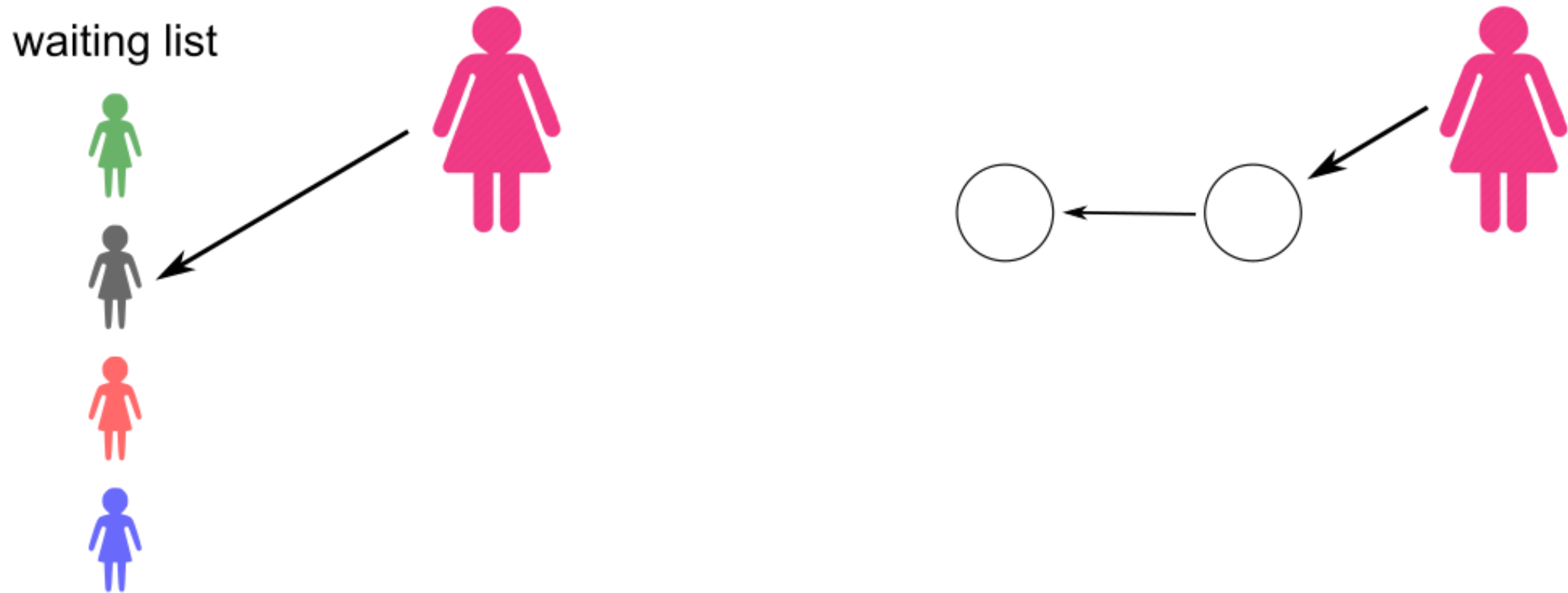
waiting list





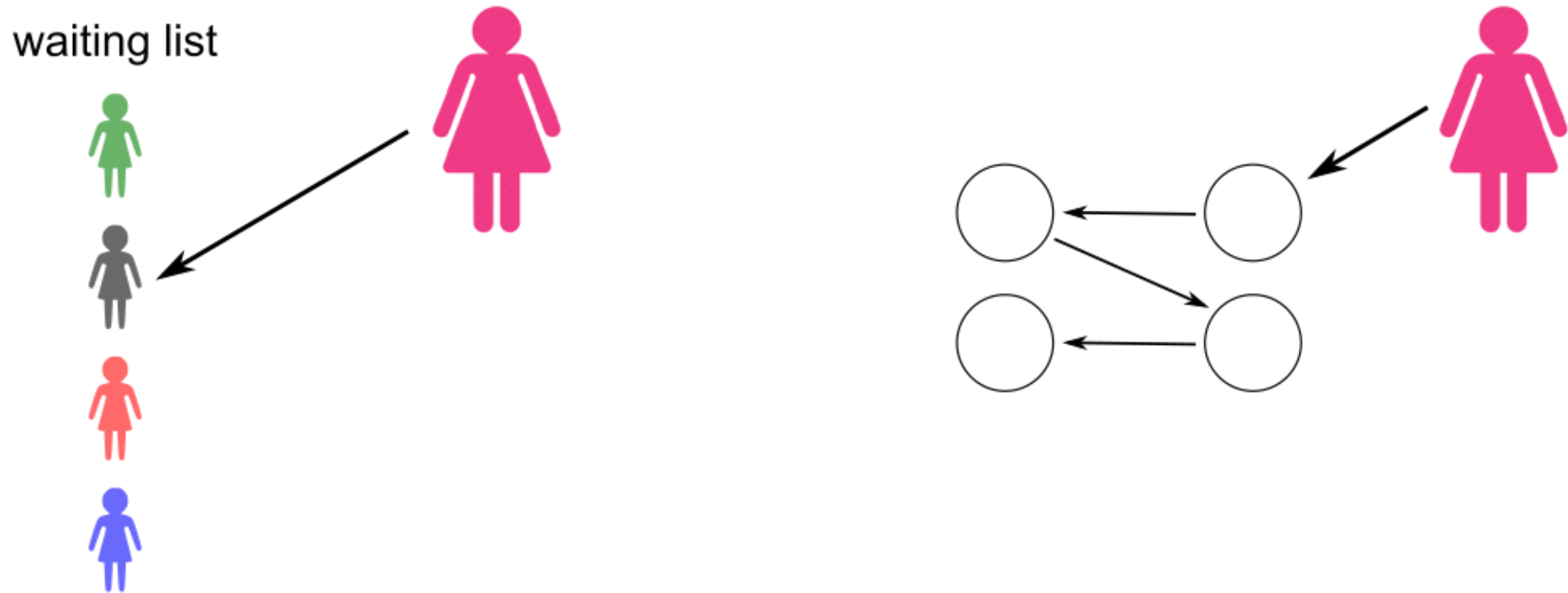
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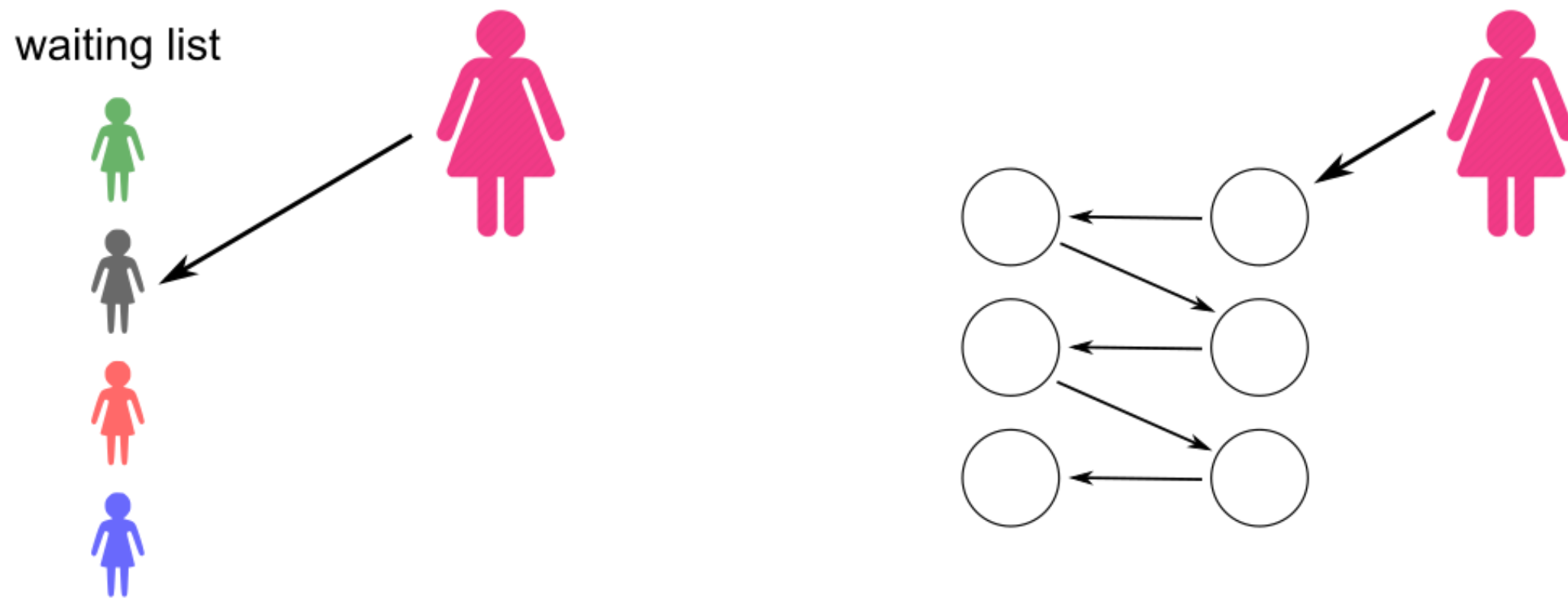
# Altruistic Donors and Chains

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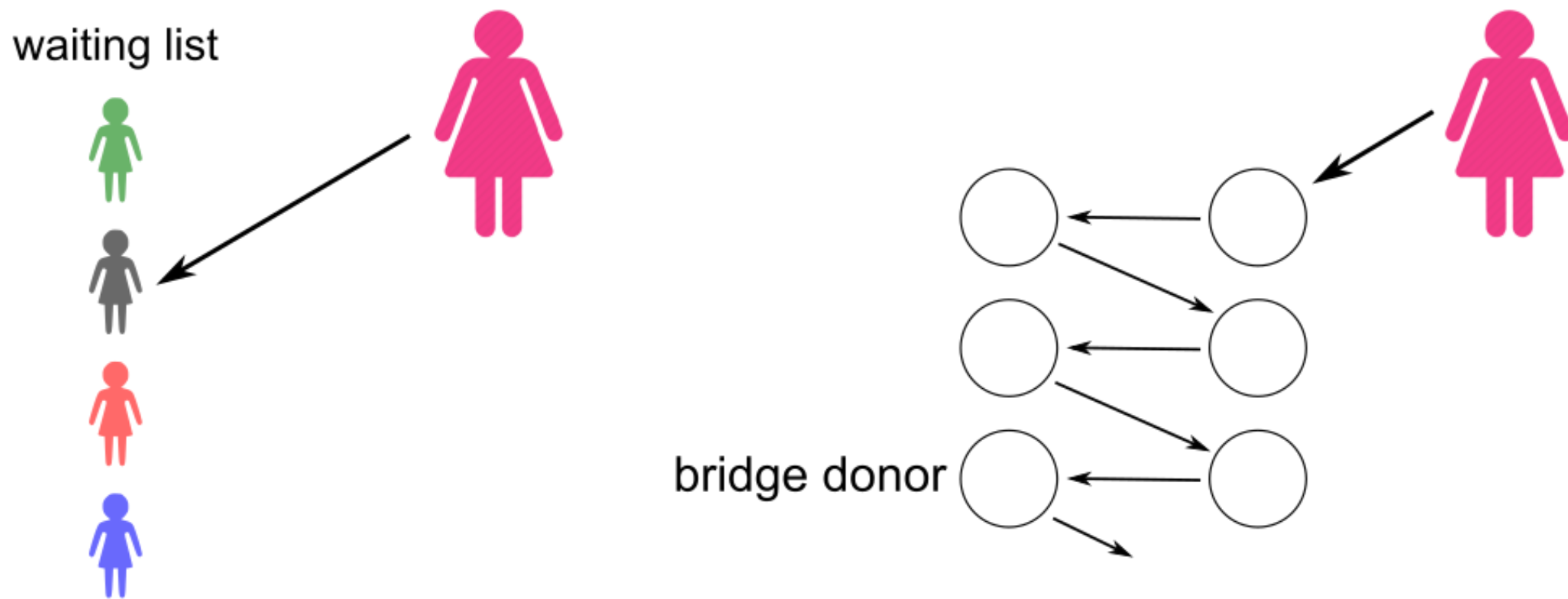
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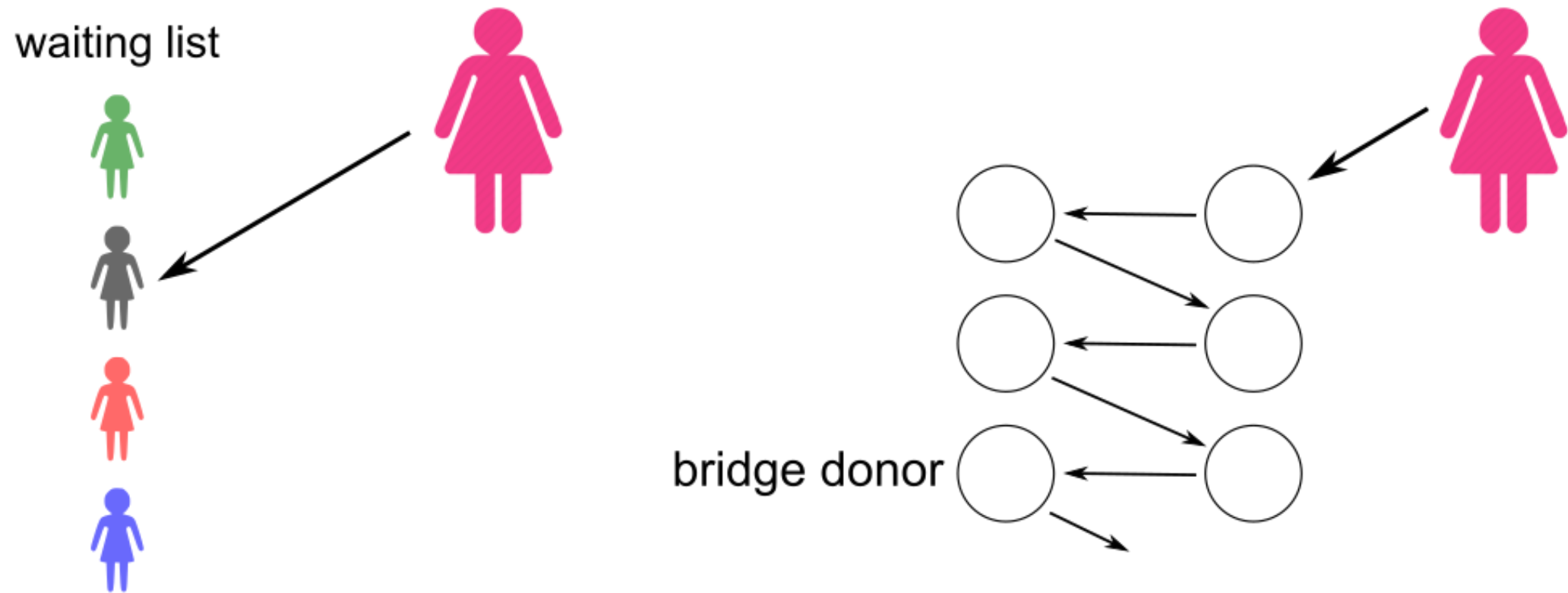
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# Altruistic Donors and Chains

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Transplants in an altruistic donor chain can be done **non-simultaneously**.



# THE KIDNEY CHAIN

How a single organ donation changed 20 lives and created the longest-running transplant chain



**MATT JONES, 30**  
Petoskey, Mich.  
First donor

donated to



**BARBARA BUNNELL, 56**  
Phoenix

married to



**RON BUNNELL, 56**  
Phoenix

donated to



**ANGELA HECKMAN, 34**  
Toledo, Ohio

daughter of



**LAURIE SARVO, 54**  
Toledo, Ohio

donated to



**REYNALDO ESPINOZA, 59**  
Germantown, Md.

donated to



**CLAUDIA ALAS, 32**  
Germantown, Md.

daughter of



**JEAN STAYLOR, 53**  
Charleston, S.C.

donated to



**RAYMOND STAYLOR, 53**  
Charleston, S.C.

married to



**AVA ROBY, 54**  
Marysville, Ohio

donated to



**GEORGE LEONER, 51**  
Chillicothe, Ohio

brother of



**LINDA JANISIESKI, 42**  
Miamisburg, Ohio

donated to



**CECILIA JANISIESKI, 71**  
Huber Heights, Ohio

mother of



**ANONYMOUS RECIPIENT**

married to



**ANONYMOUS DONOR**

donated to



**BILL CORAM, 55**  
Lincolnton, N.C.

friend of



**TIM SHAIN, 43**  
Lincolnton, N.C.

donated to



**LINLEY BLENKENSOPP, 51**  
Patchogue, N.Y.

brother of



**KURT BLENKENSOPP, 41**  
Patchogue, N.Y.

donated to



**KATHERINE MCKINNEY, 62**  
Toledo, Ohio

mother of



**HELEENA MCKINNEY, 29**  
Cincinnati  
Donor-in-waiting

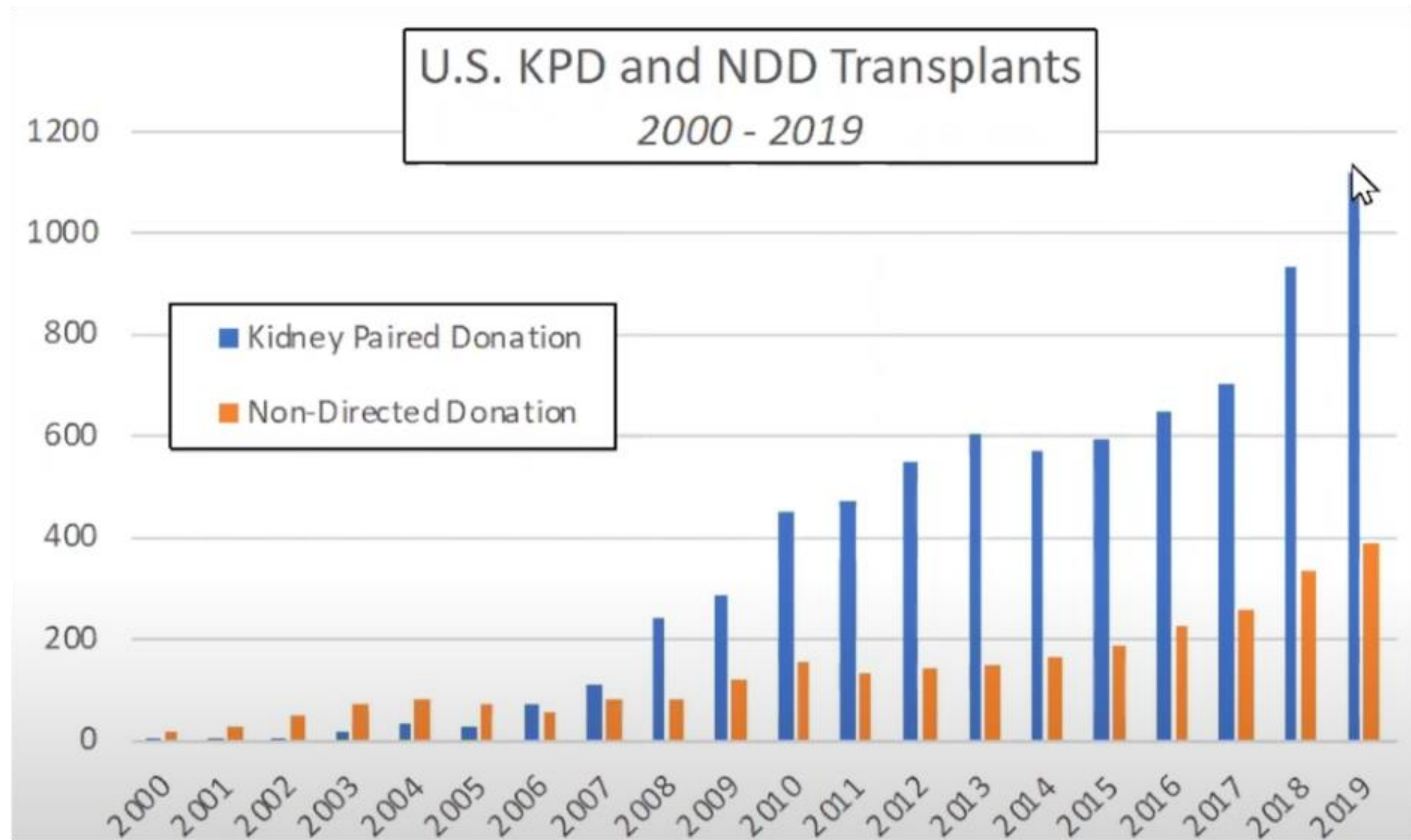
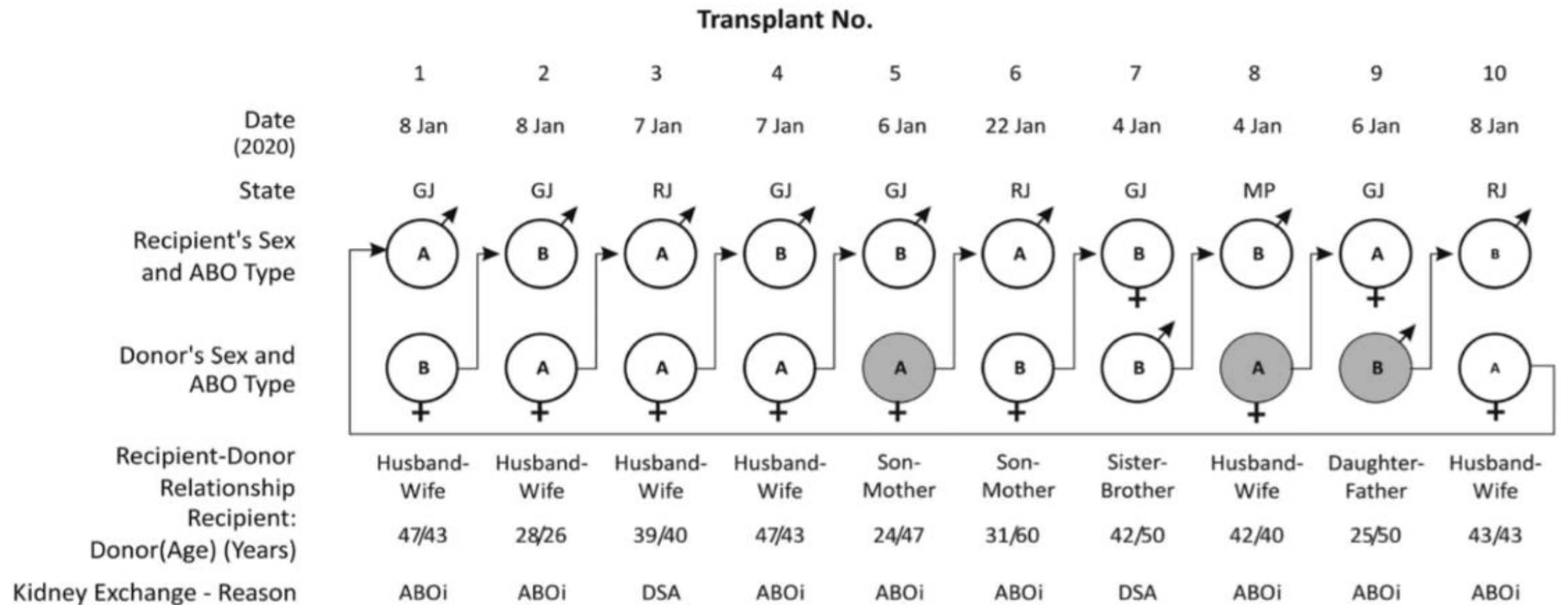


Figure source: Talk by Alvin Roth on "*Kidney Exchange: An Operations Perspective*" hosted by INFORMS Health Applications Society; June 2021



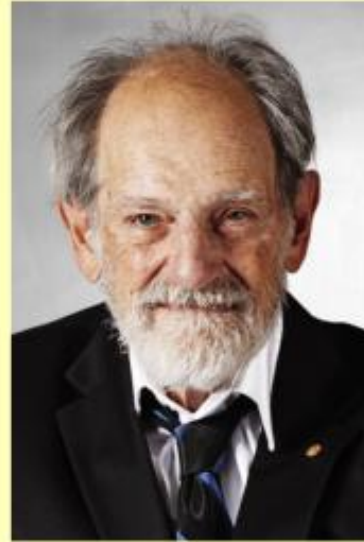
**FIGURE 1.** A nonsimultaneous 10-way kidney exchange cycle. GJ, Gujarat; MP, Madhya Pradesh; RJ, Rajasthan.



# Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012



Alvin E. Roth



Lloyd S. Shapley

*"for the theory of stable allocations  
and the practice of market design."*

# Challenges in Kidney Exchange

**Economic:** Incentives of hospitals vs maximizing overall welfare.

**Computational:** Finding a maximum matching using 3-cycles is NP-hard.

**Informational:** Patients and donors arrive sequentially.

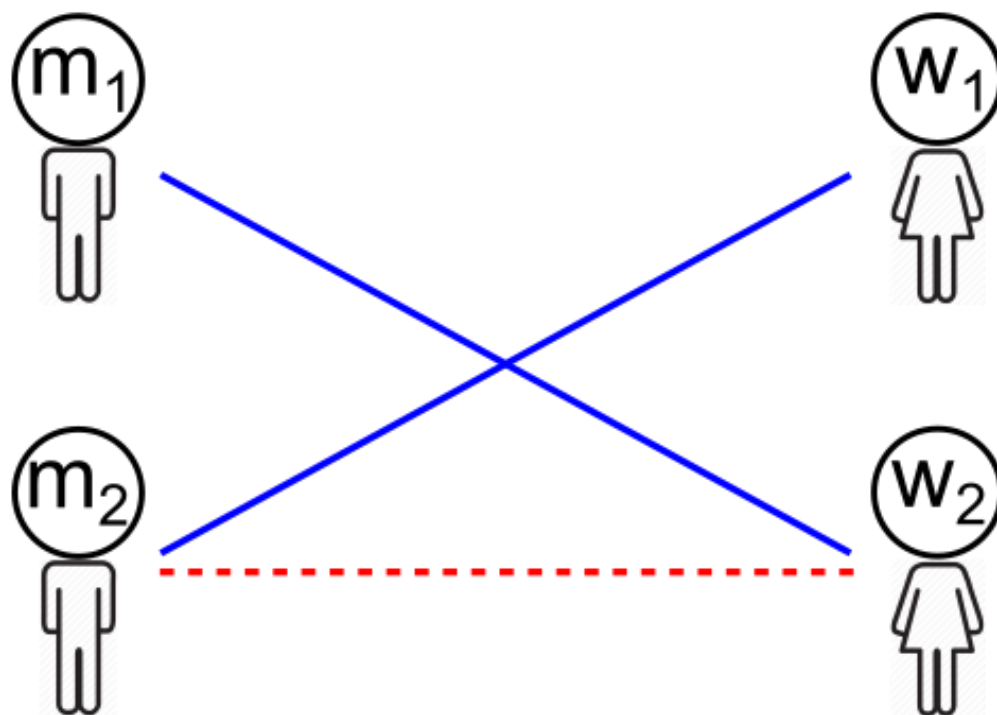
**Logistical:** Long cycles require many simultaneous surgeries.

**Legal:** Many countries do not permit altruistic donation.

**Ethical:** Is it OK to buy and sell kidneys?

# Next Time

## Stable Matchings



Reminder: **Project topics due today!**

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<https://optn.transplant.hrsa.gov/data/>
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<http://marketdesigner.blogspot.com/search/label/kidney%20exchange>

